WKST ICMAB TRANSCENDENTAL FCTS II

1. Oil is being pumped continuously from a certain oil well at a rate proportional to the amount of oil left in the well; that is, *dy/dt= ky* , where y is the amount of oil left in the well at any time t. Initially there were 1,000,000 gallons of oil in the well, and 6 years later there were 500,000 gallons remaining. It will no longer be profitable to pump oil when there are fewer than 50,000 gallons remaining.

a. Write a differential equation for y, the amount of oil remaining in the well at any time t.

b. At what rate is the amount of oil in the well decreasing when there are 600,000 gallons of oil remaining?

c. In order not to lose money

2. Let f be the function defined by f(x) = $2xe^{-x}$ for all real numbers x.

a. Write an equation of the horizontal asymptote for the graph of f.

b. Find the x-coordinate of each critical point of f. For each such x, determine whether f(x) is a relative maximum, a relative minimum, or neither.

c. For what values of x is the graph of f concave down?

d. Using the results found in parts a, b, and c, sketch the graph of y = f(x)

3. At time t, $t > 0$, the volume of a sphere is increasing at a rate proportional to the reciprocal of its radius. At $t = 0$, the radius of the sphere is 1 and at $t = 15$, the radius is 2.

a. Find the radius of the sphere as a function of t.

b. At what time t will the volume of the sphere be 27 times its volume at $t = 0$?

4. Let f be a function with $f(1) = 4$ such that for all points (x,y) on the graph of f the slope is given by

$$\frac{dy}{dx} = \frac{3x^2 + 1}{2y}$$

a) Find the slope of the graph of f at the point where x=1.

b) Where an equation for the line tangent to the graph of f at x=1 and use it to approximate f(1,2)

c) Find f(x) by solving the inseparable differential equation dy/dx=3x^2+1/2y with the initial condition f(1)=4.

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1. Oil is being pumped continuously from a certain oil well at a rate proportional to the amount of oil left in the well; that is, *dy/dt= ky* , where y is the amount of oil left in the well at any time t. Initially there were 1,000,000 gallons of oil in the well, and 6 years later there were 500,000 gallons remaining. It will no longer be profitable to pump oil when there are fewer than 50,000 gallons remaining.

Solve

a. ~~Write~~ a differential equation for y, the amount of oil remaining in the well at any time t.

$y = C \cdot e^{kt}$

$y = 1{,}000{,}000\, e^{kt}$

$\frac{1}{2} = e^{6k}$

$\ln \frac{1}{2} = 6k$

$\frac{1}{6} \ln \frac{1}{2} = k$

$y = 1{,}000{,}000\, e^{\left(\frac{1}{6} \ln \frac{1}{2}\right) t}$

$y = 1{,}000{,}000\, e^{-.1155 t}$

b. At what rate is the amount of oil in the well decreasing when there are 600,000 gallons of oil remaining?

$\frac{dy}{dt} = 600{,}000 \cdot k = -69{,}314.72$ gallons.

c. In order not to lose money, when should they stop pumping oil from the well?

$50{,}000 = 1{,}000{,}000\, e$

2. Let f be the function defined by f(x) = $2xe^{-x}$ for all real numbers x.

a. Write an equation of the horizontal asymptote for the graph of f. (calc)

$y = 0$

b. Find the x-coordinate of each critical point of f. For each such x, determine whether f(x) is a relative maximum, a relative minimum, or neither.

$-2xe^{-x} + 2e^{-x}$

$f' = 2e^{-x} - 2xe^{-x}$

$= 2e^{-x}(1 - x) \quad x = 1$

$+ \quad -$

relative max.

c. For what values of x is the graph of f concave down?

$f'' = 2e^{-x}(-1) - 2e^{-x}(1-x)$

$= 2e^{-x}(-1 - (1-x))$

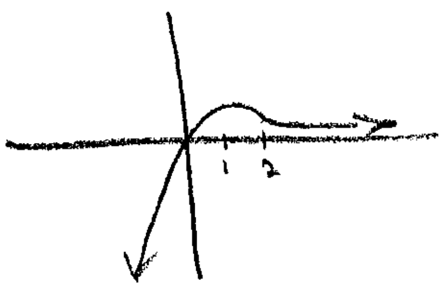
$= 2e^{-x}(-2 + x)$

$x = 2$

$- \quad +$

2

d. Using the results found in parts a, b, and c, sketch the graph of y = f(x)



Domain $(-\infty, \infty)$

Range $(-\infty, f(1)]$

3. At time t, t > 0, the volume of a sphere is increasing at a rate proportional to the reciprocal of its radius. At t = 0, the radius of the sphere is 1 and at t = 15, the radius is 2.

$\frac{dV}{dt} = \frac{k}{r}$ $V = \frac{4}{3}\pi r^3$ $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ $\frac{k}{r} = 4\pi r^2 \frac{dr}{dt}$ $k\,dt = 4\pi r^3\,dr$

a. Find the radius of the sphere as a function of t.

$\pi r^4 = kt + C$

$r^4 = \frac{kt + C}{\pi}$

$1 = \frac{C}{\pi}$ $C = \pi$

$r^4 = \frac{kt + \pi}{\pi}$

$16 = \frac{15k + \pi}{\pi}$

$16\pi = 15k + \pi$

$15\pi = 15k$

$\pi = k$

$r^4 = t + 1$

$r = \sqrt[4]{t+1}$

b. At what time t will the volume of the sphere be 27 times its volume at t = 0?

$t = 0$

$r = 1$

$V = \frac{4}{3}\pi$

$V(27) = \frac{4}{3}\pi(27)$

$36\pi = V$

$36\pi = \frac{4}{3}\pi r^3$

$3 = r$

$3 = \sqrt[4]{t+1}$

$81 = t + 1$

$80 = t$

4. Let f be a function with f(1) = 4 such that for all points (x,y) on the graph of f the slope is given by

$(1,4)$

$$\frac{dy}{dx} = \frac{3x^2+1}{2y}$$

$\int 2y\,dy = \int 3x^2 + 1\,dx$

$y^2 = x^3 + x + C$

$y^2 = x^3 + x + 14$

$y = \sqrt{x^3 + x + 14}$

a) Find the slope of the graph of f at the point where x=1.

$\frac{1}{2} \leftarrow \frac{4}{8}$

b) Where an equation for the line tangent to the graph of f at x=1 and use it to approximate f(1.2)

$y - 4 = \frac{1}{2}(x - 1)$

$y - 4 = \frac{1}{2}(1.2 - 1)$

4.1

c) Find f(x) by solving the inseparable differential equation dy/dx=3x^2+1/2y with the initial condition f(1)=4.

$f(x) = \sqrt{x^3 + x + 14}$

Find $f(1.2) = 4.114$.