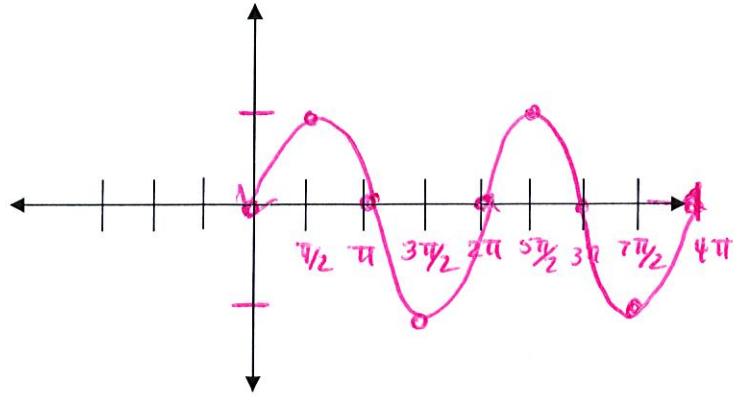
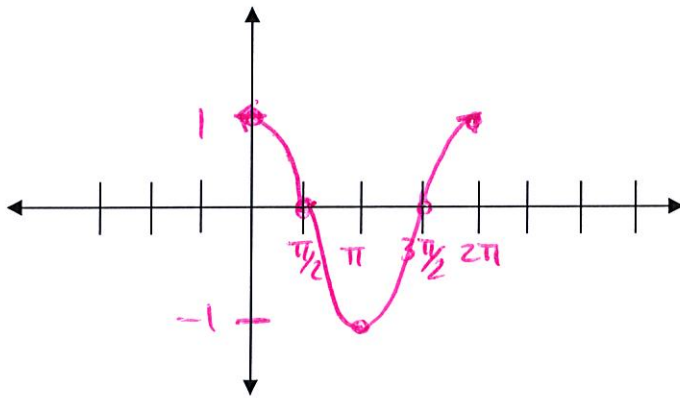


## 6.2 Graphs of Sine and Cosine Functions with Amplitude and Period Changes

### 1. Graph of $y = \sin x$



### 2. Graph of $y = \cos x$



### 3. Definition of Amplitude of Sine and Cosine Curves

The **amplitude** of  $y = a \sin x$  and  $y = a \cos x$  represents half the distance between the maximum and the minimum values of the function and is given by  $\text{Amplitude} = |a|$ .

### 4. Determine the amplitude of each, then graph.

a.  $y = \frac{1}{2} \sin x$

b.  $y = -2 \sin x$

c.  $y = \frac{3}{2} \cos x$

d.  $y = -\cos x$

### 5. Period of Sine and Cosine Functions

Let  $b$  be a positive real number. The period of  $y = a \sin bx$  and  $y = a \cos bx$  is given by  $\text{Period} = \frac{2\pi}{b}$

$|b| < 1$

horizontally stretch

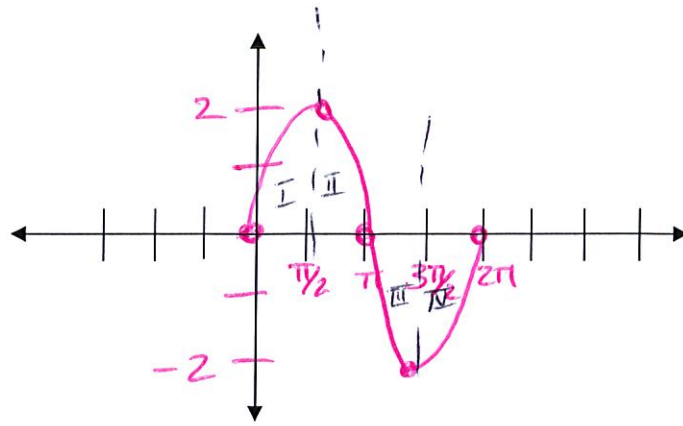
$|b| > 1$

horizontally compressed

6. Sketch the graph of each.

a.  $y = 2 \sin x$

amp = 2 per  $\frac{2\pi}{1} = 2\pi$

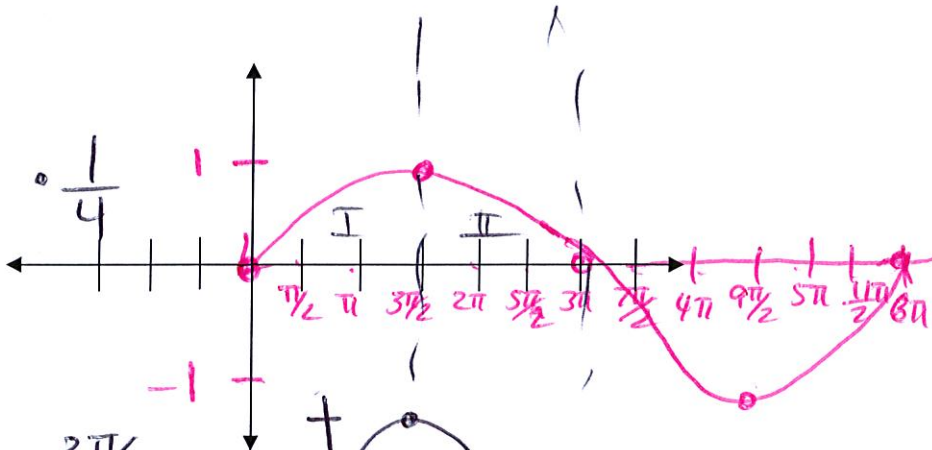


b.  $y = \sin \frac{x}{3}$

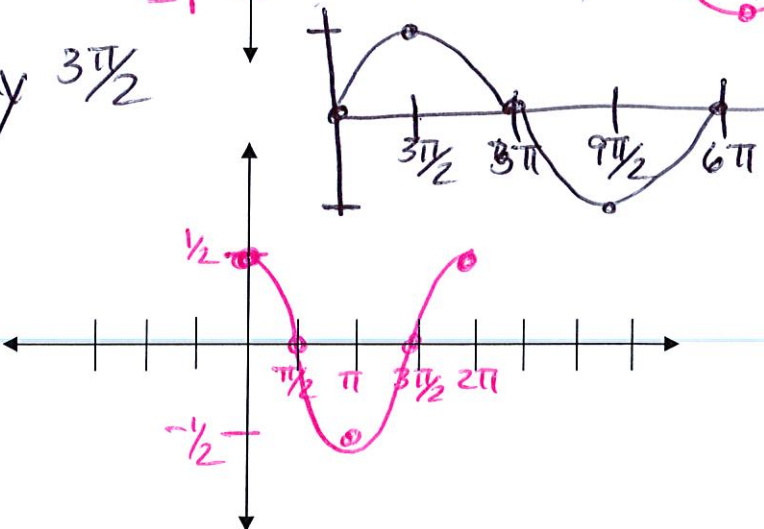
amp = 1 per  $\frac{2\pi}{1/3} = 6\pi$

x	y
0	$\sin \frac{1}{3} \cdot \frac{3\pi}{2}$
$3\pi/2$	

key pt every  $3\pi/2$



c.  $y = 2 \cos x$

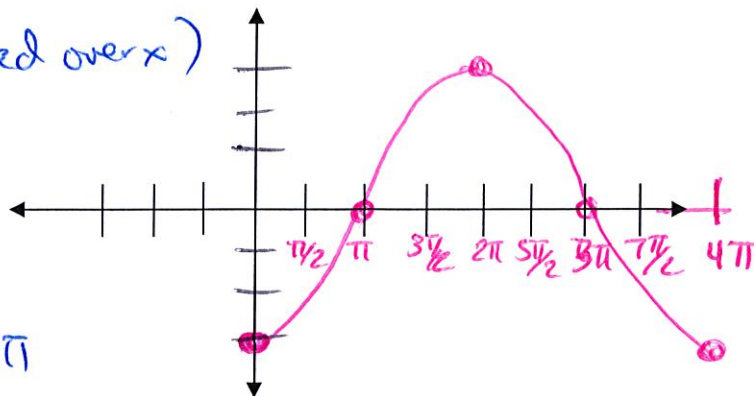


d.  $y = -3 \cos \frac{1}{2}x$  (reflected over x)

amp = 3 per  $\frac{2\pi}{1/2} = 4\pi$

horizontally stretched by 2

increment  $4\pi \cdot \frac{1}{4} = \pi$



## 6.2 Graphs of Sine and Cosine Functions with Phase Shifts.

**Phase Shift:** For  $y = a \sin b(x + c) + d$  and  $y = a \cos b(x + c) + d$ ,

The amplitude is  $|a|$ .

The period is  $\frac{2\pi}{|b|}$ .

The **phase shift** from  $y = a \sin bx$  or  $y = a \cos bx$  is  $c$  units to the left if  $c > 0$ , and  $|c|$  units to the right if  $c < 0$ .

1. Graph each of the following.

a.  $y = 4 \sin\left(2x - \frac{2\pi}{3}\right)$

$y = 4 \sin 2\left(x - \frac{\pi}{3}\right)$

amp = 4    Period =  $\frac{2\pi}{2} = \pi$

Phase: right  $\frac{\pi}{3}$

increment = Period  $\times \frac{1}{4} = \frac{\pi}{4}$

increment is  $45^\circ$  but start at  $30^\circ$

b.  $y = \frac{1}{2} \cos(4x + \pi)$

$y = \frac{1}{2} \cos 4\left(x + \frac{\pi}{4}\right)$

amp =  $\frac{1}{2}$     Period  $\frac{\pi}{2}$

Phase =  $\frac{\pi}{4}$  left

increment  $\frac{\pi}{2} \cdot \frac{1}{4} = \frac{\pi}{8}$

$\frac{\pi}{8}$  goes into  $\frac{\pi}{4}$  so scale is  $\frac{\pi}{8}$

Vertical Shifts: The **vertical shift** from  $y = a \sin bx$  or  $y = a \cos bx$  is  $d$  up if  $d > 0$ , and  $d$  units down if  $d < 0$ .

2. Graph each of the following:

a.  $y = 2 \cos(x) - 1$

b.  $y = 3 \sin(x + \pi) + 2$

