AP® CALCULUS AB/CALCULUS BC 2015 SCORING GUIDELINES

Question 1

The rate at which rainwater flows into a drainpipe is modeled by the function R, where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \le t \le 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \le t \le 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

(a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \le t \le 8$?

(b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.

(c) At what time t, $0 \le t \le 8$, is the amount of water in the pipe at a minimum? Justify your answer.

(d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

(a) $\int_0^8 R(t)\,dt = 76.570$

$2: \begin{cases} 1: \text{integrand} \\ 1: \text{answer} \end{cases}$

(b) $R(3) - D(3) = -0.313632 < 0$

Since $R(3) < D(3)$, the amount of water in the pipe is decreasing at time $t = 3$ hours.

$2: \begin{cases} 1: \text{considers } R(3) \text{ and } D(3) \\ 1: \text{answer and reason} \end{cases}$

(c) The amount of water in the pipe at time t, $0 \le t \le 8$, is

$30 + \int_0^t [R(x) - D(x)]\,dx.$

$R(t) - D(t) = 0 \Rightarrow t = 0,\ 3.271658$

t	Amount of water in the pipe
0	30
3.271658	27.964561
8	48.543686

The amount of water in the pipe is a minimum at time $t = 3.272$ (or 3.271) hours.

$3: \begin{cases} 1: \text{considers } R(t) - D(t) = 0 \\ 1: \text{answer} \\ 1: \text{justification} \end{cases}$

(d) $30 + \int_0^w [R(t) - D(t)]\,dt = 50$

$2: \begin{cases} 1: \text{integral} \\ 1: \text{equation} \end{cases}$

© 2015 The College Board.
Visit the College Board on the Web: www.collegeboard.org.

Question 2

The rate at which people enter an auditorium for a rock concert is modeled by the function R given by $R(t) = 1380t^2 - 675t^3$ for $0 \le t \le 2$ hours; $R(t)$ is measured in people per hour. No one is in the auditorium at time $t = 0$, when the doors open. The doors close and the concert begins at time $t = 2$.

(a) How many people are in the auditorium when the concert begins?

(b) Find the time when the rate at which people enter the auditorium is a maximum. Justify your answer.

(c) The total wait time for all the people in the auditorium is found by adding the time each person waits, starting at the time the person enters the auditorium and ending when the concert begins. The function w models the total wait time for all the people who enter the auditorium before time t. The derivative of w is given by $w'(t) = (2 - t)R(t)$. Find $w(2) - w(1)$, the total wait time for those who enter the auditorium after time $t = 1$.

(d) On average, how long does a person wait in the auditorium for the concert to begin? Consider all people who enter the auditorium after the doors open, and use the model for total wait time from part (c).

(a) $\int_0^2 R(t)\,dt = 980$ people

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) $R'(t) = 0$ when $t = 0$ and $t = 1.36296$

The maximum rate may occur at 0, $a = 1.36296$, or 2.

$R(0) = 0$
$R(a) = 854.527$
$R(2) = 120$

The maximum rate occurs when $t = 1.362$ or 1.363.

$3 : \begin{cases} 1 : \text{considers } R'(t) = 0 \\ 1 : \text{interior critical point} \\ 1 : \text{answer and justification} \end{cases}$

(c) $w(2) - w(1) = \int_1^2 w'(t)\,dt = \int_1^2 (2 - t)R(t)\,dt = 387.5$

The total wait time for those who enter the auditorium after time $t = 1$ is 387.5 hours.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) $\frac{1}{980} w(2) = \frac{1}{980} \int_0^2 (2 - t)R(t)\,dt = 0.77551$

On average, a person waits 0.775 or 0.776 hour.

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

© 2009 The College Board. All rights reserved.
Visit the College Board on the Web: www.collegeboard.com.

3

Question 1

A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 60-day period. The height of water in the can is modeled by the function S, where $S(t)$ is measured in millimeters and t is measured in days for $0 \le t \le 60$. The rate at which the height of the water is rising in the can is given by $S'(t) = 2\sin(0.03t) + 1.5$.

(a) According to the model, what is the height of the water in the can at the end of the 60-day period?

(b) According to the model, what is the average rate of change in the height of water in the can over the 60-day period? Show the computations that lead to your answer. Indicate units of measure.

(c) Assuming no evaporation occurs, at what rate is the volume of water in the can changing at time $t = 7$? Indicate units of measure.

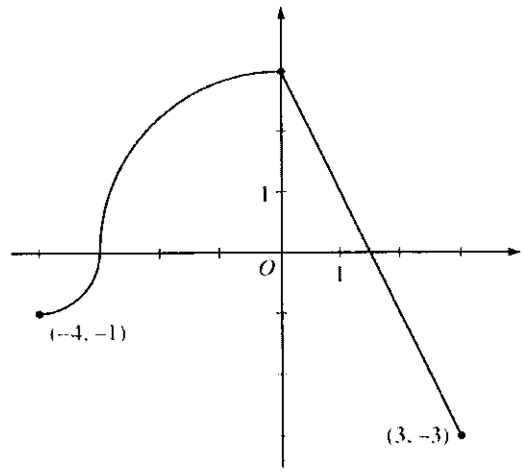
(d) During the same 60-day period, rain on Monsoon Mountain accumulates in a can identical to the one in Stormville. The height of the water in the can on Monsoon Mountain is modeled by the function M, where $M(t) = \frac{1}{400}\left(3t^3 - 30t^2 + 330t\right)$. The height $M(t)$ is measured in millimeters, and t is measured in days for $0 \le t \le 60$. Let $D(t) = M'(t) - S'(t)$. Apply the Intermediate Value Theorem to the function D on the interval $0 \le t \le 60$ to justify that there exists a time t, $0 < t < 60$, at which the heights of water in the two cans are changing at the same rate.

(a) $S(60) = \int_0^{60} S'(t)\,dt = 171.813$ mm	$3: \begin{cases} 1: \text{limits} \\ 1: \text{integrand} \\ 1: \text{answer} \end{cases}$
(b) $\dfrac{S(60) - S(0)}{60} = 2.863$ or 2.864 mm/day	1 : answer
(c) $V(t) = 100\pi S(t)$ $V'(7) = 100\pi S'(7) = 602.218$ The volume of water in the can is increasing at a rate of $602.218\ \text{mm}^3/\text{day}$.	$2: \begin{cases} 1: \text{relationship between } V \text{ and } S \\ 1: \text{answer} \end{cases}$
(d) $D(0) = -0.675 < 0$ and $D(60) = 69.37730 > 0$ Because D is continuous, the Intermediate Value Theorem implies that there is a time t, $0 < t < 60$, at which $D(t) = 0$. At this time, the heights of water in the two cans are changing at the same rate.	$2: \begin{cases} 1: \text{considers } D(0) \text{ and } D(60) \\ 1: \text{justification} \end{cases}$
	1 : units in (b) or (c)

© 2011 The College Board.
Visit the College Board on the Web: www.collegeboard.org.

Question 4

The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above.



Graph of f

Let $g(x) = 2x + \int_0^x f(t)\,dt$.

(a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

(b) Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \le x \le 3$. Justify your answer.

(c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.

(d) Find the average rate of change of f on the interval $-4 \le x \le 3$. There is no point c, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

(a) $g(-3) = 2(-3) + \int_0^{-3} f(t)\,dt = -6 - \dfrac{9\pi}{4}$

$g'(x) = 2 + f(x)$

$g'(-3) = 2 + f(-3) = 2$

$3 : \begin{cases} 1 : g(-3) \\ 1 : g'(x) \\ 1 : g'(-3) \end{cases}$

(b) $g'(x) = 0$ when $f(x) = -2$. This occurs at $x = \dfrac{5}{2}$.

$g'(x) > 0$ for $-4 < x < \dfrac{5}{2}$ and $g'(x) < 0$ for $\dfrac{5}{2} < x < 3$.

Therefore g has an absolute maximum at $x = \dfrac{5}{2}$.

$3 : \begin{cases} 1 : \text{considers } g'(x) = 0 \\ 1 : \text{identifies interior candidate} \\ 1 : \text{answer with justification} \end{cases}$

(c) $g''(x) = f'(x)$ changes sign only at $x = 0$. Thus the graph of g has a point of inflection at $x = 0$.

1 : answer with reason

(d) The average rate of change of f on the interval $-4 \le x \le 3$ is $\dfrac{f(3) - f(-4)}{3 - (-4)} = -\dfrac{2}{7}$.

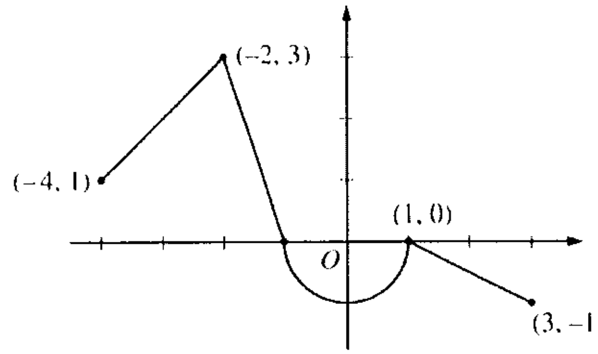
To apply the Mean Value Theorem, f must be differentiable at each point in the interval $-4 < x < 3$. However, f is not differentiable at $x = -3$ and $x = 0$.

$2 : \begin{cases} 1 : \text{average rate of change} \\ 1 : \text{explanation} \end{cases}$

© 2011 The College Board.
Visit the College Board on the Web: www.collegeboard.org.

Question 3

Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt$.



Graph of f

(a) Find the values of $g(2)$ and $g(-2)$.

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

(c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

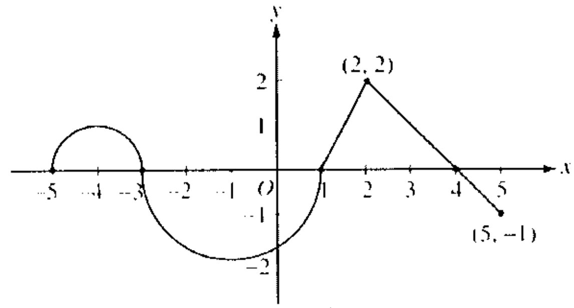
(a) $g(2) = \int_1^2 f(t)\,dt = -\frac{1}{2}(1)\left(\frac{1}{2}\right) = -\frac{1}{4}$ $g(-2) = \int_1^{-2} f(t)\,dt = -\int_{-2}^{1} f(t)\,dt$ $= -\left(\frac{3}{2} - \frac{\pi}{2}\right) = \frac{\pi}{2} - \frac{3}{2}$	$2: \begin{cases} 1: g(2) \\ 1: g(-2) \end{cases}$
(b) $g'(x) = f(x) \Rightarrow g'(-3) = f(-3) = 2$ $g''(x) = f'(x) \Rightarrow g''(-3) = f'(-3) = 1$	$2: \begin{cases} 1: g'(-3) \\ 1: g''(-3) \end{cases}$
(c) The graph of g has a horizontal tangent line where $g'(x) = f(x) = 0$. This occurs at $x = -1$ and $x = 1$. $g'(x)$ changes sign from positive to negative at $x = -1$. Therefore, g has a relative maximum at $x = -1$. $g'(x)$ does not change sign at $x = 1$. Therefore, g has neither a relative maximum nor a relative minimum at $x = 1$.	$3: \begin{cases} 1: \text{considers } g'(x) = 0 \\ 1: x = -1 \text{ and } x = 1 \\ 1: \text{answers with justifications} \end{cases}$
(d) The graph of g has a point of inflection at each of $x = -2$, $x = 0$, and $x = 1$ because $g''(x) = f'(x)$ changes sign at each of these values.	$2: \begin{cases} 1: \text{answer} \\ 1: \text{explanation} \end{cases}$

© 2012 The College Board.
Visit the College Board on the Web: www.collegeboard.org.

AP® CALCULUS AB
2007 SCORING GUIDELINES (Form B)

Question 4

Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.



Graph of f'

(a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.

(b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.

(c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.

(d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \le x \le 5$. Explain your reasoning.

(a) $f'(x) = 0$ at $x = -3,\ 1,\ 4$

f' changes from positive to negative at -3 and 4.

Thus, f has a relative maximum at $x = -3$ and at $x = 4$.

2 : { 1 : x-values; 1 : justification }

(b) f' changes from increasing to decreasing, or vice versa, at $x = -4,\ -1$, and 2. Thus, the graph of f has points of inflection when $x = -4,\ -1$, and 2.

2 : { 1 : x-values; 1 : justification }

(c) The graph of f is concave up with positive slope where f' is increasing and positive: $-5 < x < -4$ and $1 < x < 2$.

2 : { 1 : intervals; 1 : explanation }

(d) Candidates for the absolute minimum are where f' changes from negative to positive (at $x = 1$) and at the endpoints ($x = -5, 5$).

$$f(-5) = 3 + \int_1^{-5} f'(x)\,dx = 3 - \frac{\pi}{2} + 2\pi > 3$$

$$f(1) = 3$$

$$f(5) = 3 + \int_1^{5} f'(x)\,dx = 3 + \frac{3 \cdot 2}{2} - \frac{1}{2} > 3$$

The absolute minimum value of f on $[-5, 5]$ is $f(1) = 3$.

3 : { 1 : identifies $x = 1$ as a candidate; 1 : considers endpoints; 1 : value and explanation }

© 2007 The College Board. All rights reserved.
Visit apcentral.collegeboard.com (for AP professionals) and www.collegeboard.com/apstudents (for students and parents).