

Day 3:

Definition of Increasing and Decreasing Functions

A function f is increasing on an interval if for any two numbers a and b in the interval, $a < b$ implies $f(a) < f(b)$.

A function f is decreasing on an interval if for any two numbers a and b in the interval, $a < b$ implies $f(a) > f(b)$.

Let f be a function that is continuous on the closed $[a, b]$ and differentiable on the open interval (a, b) .

1. If $f'(x) > 0$ for all x in (a, b) , then f is increasing on $[a, b]$.

2. If $f'(x) < 0$ for all x in (a, b) , then f is decreasing on $[a, b]$.

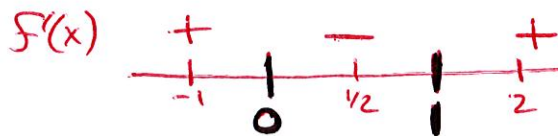
3. If $f'(x) = 0$ for all x in (a, b) , then f is constant on $[a, b]$.

Ex 1: Find the open intervals on which $f(x) = x^3 - \frac{3}{2}x^2$ is increasing or decreasing. (Graph)

$$f'(x) = 3x^2 - 3x$$

$$3x(x-1)$$

Critical Points $x=0$ $x=1$



$f(x)$ is increasing $(-\infty, 0)$
 $(1, \infty)$

decreasing $(0, 1)$

Ex 2: Find the open intervals on which $f(x) = \frac{x^2}{4x+4}$ is increasing or decreasing.

$$f'(x) = \frac{(4x+4)2x - x^2 \cdot 4}{(4x+4)^2}$$

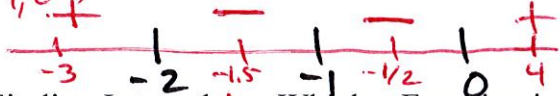
$x \neq -1$

inc $(-\infty, -2) \cup (0, \infty)$

$$(4x+4)^2 = \frac{4x(x+2)}{(4x+4)^2} = 0$$

dec $(-2, -1) \cup (-1, 0)$

Critical Points $x = -2, -1, 0$



Guidelines for Finding Intervals on Which a Function is Increasing or Decreasing

1. Locate the Critical Points on the interval (a, b) , use these numbers to determine test intervals.
2. Determine the sign of $f'(x)$ at one test value in each of the intervals.
3. Determine whether f is increasing or decreasing.

The First Derivative Test:

Let c be a critical number of a function f that is continuous on an open interval I containing c .

If f is differentiable on the interval, except possibly at c , then $f(c)$ can be classified as follows.

1. If $f'(x)$ changes from negative to positive at c , then $f(c)$ is a relative minimum of f .
2. If $f'(x)$ changes from positive to negative at c , then $f(c)$ is a relative maximum of f .
3. If $f'(x)$ does not change sign at c , then $f(c)$ is neither a relative minimum nor a relative maximum.

Find the relative extrema of the functions below, if they exist.

Ex 3: $f(x) = -x^3 - 3x^2 - 1$

$$f'(x) = -3x(x+2)$$

Critical Points $x=0$ $x=-2$



decreasing $(-\infty, -2) \cup (0, \infty)$
increasing $(-2, 0)$

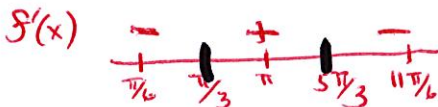
Relative min $(-2, -5)$ max $(0, -1)$

Ex 5: $f(x) = x - 2\sin(x); (0, 2\pi)$

$$f'(x) = 1 - 2\cos(x)$$

$$\cos x = 1/2$$

Critical Points $x = \pi/3, 5\pi/3$



Relative min $(\pi/3, \pi/3 - \sqrt{3})$ max $(5\pi/3, 5\pi/3 + \sqrt{3})$

Ex 7: $f(x) = \cos^2(x) - 2\sin(x); (0, 2\pi)$

$$f'(x) = 2\cos(x) \cdot (-\sin x) - 2\cos x$$

$$= 2\cos(x) (-\sin x - 1)$$

Critical Points $x = \pi/2, 3\pi/2$



relative min $(\pi/2, -2)$ relative max $(3\pi/2, 2)$

Ex 4: $f(x) = \frac{-2}{x^2 - 4}$ Critical Points $x \neq \pm 2$

$$f'(x) = \frac{-2(2x)}{(x^2 - 4)^2} \quad x = 0$$



decreasing $(-\infty, -2) \cup (-2, 0)$
increasing $(0, 2) \cup (2, \infty)$

Relative min $(0, 1/2)$

Ex 6: $f(x) = (x^2 - 4)^{1/3}$

$$f'(x) = \frac{1}{3}(x^2 - 4)^{-2/3} \cdot 2x = \frac{2x}{3\sqrt[3]{(x^2 - 4)^2}}$$

Critical Points $x=0$ $x=\pm 2$



Relative min $(0, -4^{1/3})$

Ex 8: $f(x) = x^4 - 4x^3$

$$f'(x) = 4x^3 - 12x^2 = 4x^2(x - 3)$$

Critical Points $x=0$ $x=3$



Relative Min $(3, -27)$