Precalculus Final Review

1. Given the following values, evaluate (if possible) the other four trigonometric functions using the fundamental trigonometric identities or triangles

$$\csc\theta = -\frac{5}{3}, \tan\theta = \frac{3}{4}$$

2. Simplify: $\frac{1}{\cot\theta} + \frac{1}{\tan\theta}$

3. Simplify: $\frac{\cos^4\theta - \sin^4\theta}{\cos^2\theta - \sin^2\theta}$

4. Factor and simplify: $\cos^2 x - \sin^2 x \cos^2 x$

5. Verify the identity: $(\tan^2 x + 1)(\cos^2 x - 1) = -\tan^2 x$

6. Verify the identity: cotx + tanx = cscx secx

7. Verify the identity: $\frac{\sec x - \cos x}{\tan x} = \sin x$

8. Verify the identity: $\frac{\cos x \csc x}{\cot^2 x} = \tan x$

9. Find all solutions: cosx – 1 = 0

10. Find all solutions: $\sin x + \sqrt{2} = -\sin x$

11. Find all solutions in the interval $[0, 2\pi)$: $\cot^2 x - 1 = 0$

12. Find all solutions in the interval $[0, 2\pi)$: $6\cos^2 x - 5 \sin x - 2 = 0$

13.Evaluate: $\tan 165^\circ$ (use the fact that 165 = 210 – 45)

14. Evaluate: $\cos \frac{5\pi}{12}$ (use the fact that $\frac{5\pi}{12} = \frac{\pi}{4} + \frac{\pi}{6}$)

15. Simplify sin8xcos3x + cos8xsin3x

16. Given $\sin u = \frac{7}{25}$, $\frac{\pi}{2} < u < \pi$ and $\cos v = \frac{4}{5}$, $\frac{3\pi}{2} < v < 2\pi$, find cos(u + v).

17. Find all solutions in the interval $[0, 2\pi]$: sin2x + sinx = 0

18. Find the exact value of cos2u using a double angle formula:

$\cos u = -\frac{2}{3}$, $\frac{\pi}{2} < u < \pi$

1. A vector **v** has initial point (-2, 1) and terminal point (7, 6).

a. Find its component form.

b. Determine its magnitude.

c. Find its direction.

2. Given **u** = 3**i** – 2**j**, **w** = 9**i** + 5**j**, and **v** = ½**u** + 4**w**, find **v**.

3. A vector **v** has magnitude 6 and direction $\theta = 210^{o}$. Find its component form.

4. Given **v** of magnitude 50 and direction $\theta_v = 315^{o}$, and **w** of magnitude 20 and direction $\theta_w = 210^{o}$, find **v** + **w**. Write the answer in component form.

5. Find the unit vector in the direction of **v** = 4**i** – 3**j**. Express the answer in linear form.

6. Find the component form of **u**: $\|u\| = 50, \theta_u = 330^{o}$.

7. Given **v** = $\langle 5, -2 \rangle$ and **w** = $\langle 6,1 \rangle$, find the angle between **v** and **w**.

8. Given **u** = 5**i** – 2**j** and **v** = -3**i** + **j**, find $u \bullet v$.

9. Find the angle between the vectors **v** = 2i + 3j and **u** = 7i – j.

10. Find $u \bullet v$ if $\|u\|$=7 and $\|v\|$=12, and the angle between u and v is $\frac{\pi}{4}$.

11. Determine whether the vectors are orthogonal, parallel, or neither.

a. $\vec{v}$ = -2i + j, $\vec{w}$ = I + 2j

b. $\vec{v}$ = -4i + j, $\vec{w}$ = -12i + 3j

c. $\vec{v}$ = -I + 2j, $\vec{w}$ = -2

1. Identify the conic represented by the equation below and find its **center.**
   Sketch the graph.

$$x^2 + 4y^2 - 2x - 16y + 1 = 0$$

2. Write the equation of the circle that passes through (4, 1) and has its center at (4, 3).

3. Eliminate the parameter and find a corresponding rectangular equation.
   Sketch the curve.

$$x = 3t - 1,\ y = 2t + 1$$

4. Eliminate the parameter and find a corresponding rectangular equation. Sketch the curve.

$$x = 3t^2,\ y = 2t + 1$$

5. Eliminate the parameter and find a corresponding rectangular equation. Sketch the curve.

$$x = 3t + 1,\ y = 2t$$

6. Eliminate the parameter and find a corresponding rectangular equation.

$$x = 4 + 2\cos\theta,\ y = -1 + \sin\theta.$$

7. Eliminate the parameter and find a corresponding rectangular equation. Sketch the curve.

$$x = 2\cos\theta,\ y = \cos^2\theta$$

8. Find a set of parametric equations to represent the graph of $y = (x - 1)^2$ given the parameter $t = x - 1$.

9. Find a set of parametric equations to represent the graph of $y = -\frac{2}{3}x + 3$ given the parameter $t = x$.

10. Find a set of parametric equations to represent the graph of $y = (x-2)^2 + 1$ given the parameter $t = x - 2$.

11. If an object moves according to the parametric equations $x = t^3$, $y = \frac{1}{2}t^2$, sketch a graph of its motion for $-3 \leq t \leq 3$.

12. If an object moves according to the parametric equations $x = 2t$, $y = 3t^2$, sketch a graph of its motion for $0 \leq t \leq 3$.

13. Convert from polar to rectangular coordinates. $(2, \frac{3\pi}{4})$

14. Convert from rectangular to polar coordinates. (-6, 0).

15. Convert from polar to rectangular coordinates. $(8, \frac{7\pi}{6})$.

16. Convert from polar to rectangular coordinates. $(-6, \frac{3\pi}{2})$.

17. Convert from rectangular to polar coordinates. (0, -4).

18. Plot the following points whose polar coordinates are

a. $\left(-3, \frac{5\pi}{6}\right)$

b. $\left(2, -\frac{3\pi}{4}\right)$ *Label each point on the polar coordinate system.

c. $\left(-4, \frac{-\pi}{3}\right)$

19. Find another set of polar coordinates that represent the point $\left(4, \frac{\pi}{2}\right)$.

20. Find three sets of polar coordinates that represent the point (-2, -$\frac{5\pi}{6}$).

21. Change from polar to rectangular equation. $r = 4\sin\theta$

22. Change the rectangular equation to polar form. $x^2 + y^2 - 2x = 0$

23. Change from polar to rectangular equation. Sketch the graph. $r = 2\cos\theta$

24. Change from polar to rectangular equation. Sketch the graph. $r\sin\theta = -3$

25. Change the rectangular equation to polar form. $x^2 + y^2 + 2x + 5y = 0$

26. Graph and identify the special type of polar graph.

a. $r = 5 - 5\sin\theta$

b. $r = 6\sin 2\theta$

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c. $r = 3\sin 4\theta$

d. $r^2 = 9\sin 2\theta$

e. $r = 1 - 2\sin\theta$

1. Find the vertex of the parabola:

$$y^2 + 6y + 9 = 16 - 16x$$

2. Find the vertex of the parabola:

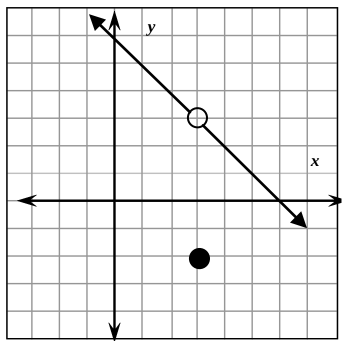
$$x^2 - 2x + 1 = 8y - 16$$

3. Find the focus of the parabola:

$$y^2 + 6y + 9 = 16 - 16x$$

Use the graph of the function *f* to determine the given limit.

4. $\lim_{x \to 3} f(x)$ [A] 6 [B] -4 [C] 3 [D] does not exist



5. $\lim_{x \to 5} f(x)$

6. Use a calculator to find $\lim_{x \to 0} \frac{2\cos(5x) - 2}{25x^2}$

[A] 1 [B] -1 [C] $\frac{2}{25}$ [D] none of these

7. Complete the table and use the result to estimate the given limit.

$$\lim_{x \to 0} \frac{\sqrt{x+15} - \sqrt{15}}{x}$$

8. Find $\lim_{x \to -8} f(x)$ for: A. *f*(x) = 3

a. 3 b. -5 c. -8 d. 8

B. *f*(x) = x

a. 0 b. -8 c. 8 d. 3