Writing an equation given the zeros:

Ex: Find a cubic equation with integral coefficients that has 2 and 3 – i as roots.

$3-i$ $(3+i)(3-i)$

$3+i$

$6 = -\frac{b}{a}$ $10 = \frac{c}{a}$

$x^2-6x+10$

$(x-2)(x^2-6x+10)$

$x^3-6x^2+10x-2x^2+12x-20$

$y = x^3-8x^2+22x-20$

Ex: Given {4, -1, 3i} write an equation with integral coefficients.

$3i$ $3i \cdot -3i$ $(x-4)(x+1)$

$-3i$

0 9 $x^2+x-4x-4$

x^2+9 x^2-3x-4

	x^2	$-3x$	-4
x^2	x^4	$-3x^3$	$-4x^2$
9	$9x^2$	$-27x$	-36

$y = x^4-3x^3+5x^2-27x-36$

Descartes' Rule Of Signs:

If $p(x)$ is a polynomial with real coefficients, then

a) the number of positive real roots is either equal to the number of variations of sign of p(x) or is less than this number by a positive even integer.

Example: $p(x) = x^4 + 8x^3 - 2x^2 - 3x + 8$

There are 2 variations in sign $\therefore$ there are 2 or 0 positive real roots.

b) the number of negative real roots is either equal to the number of variations of sign of p(-x) or less than this number by a positive even integer.

Example: $p(-x) = (-x)^4 + 8(-x)^3 - 2(-x)^2 - 3(-x) + 8 = x^4 - 8x^3 - 2x^2 + 3x + 8$

There are 2 variations in sign $\therefore$ there are 2 or 0 negative real roots.

Possibilities for types of roots:

+	-	imag.
2	2	0
2	0	2
0	2	2
0	0	4

All rows must add up to 4 (degree of polynomial)

Descartes Rules of Signs

Ex: $g(x) = 3x^4 - x^3 + 8x^2 + x - 7$

3 or 1 pos

$g(-x) = 3x^4 + x^3 + 8x^2 - x - 7$

1 neg

P	N	I	
3	1	0	= 4 ← degree
1	1	2	

Ex: $f(x) = x^5 - x^3 - x + 1$

0 doesn't count

2 or 0 pos.

$f(-x) = -x^5 + x^3 + x + 1$

1 neg

P	N	I	
2	1	2	
0	1	4	= 5 ← degree

Ex: $f(x) = x^3 + 2x^2 - 5x - 6$

1 pos

$f(-x) = -x^3 + 2x^2 + 5x - 6$

2 or 0 neg

P	N	I	
1	2	0	
1	0	2	= 3 ← degree

Ex: $f(h) = h^3 + 7h^2 - 36$

1 pos

$f(-h) = -h^3 + 7h^2 - 36$

2 or 0 neg

P	N	I	
1	2	0	
1	0	2	= 3