Day 8 Notes:

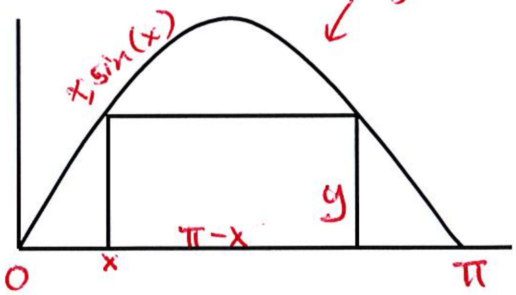
1. A rectangle is to be inscribed under one arc of the sine curve. What is the largest area the rectangle can have, and what dimensions give that area?

Symmetric

height base

$$\text{Area} = \sin x\,(\pi - x)$$

$$= \pi \sin x - x \sin x$$

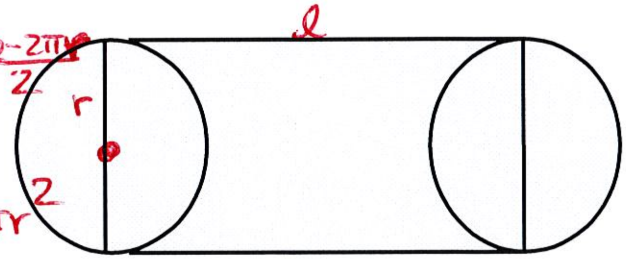


2. An indoor physical fitness room consists of a rectangular region with a semicircle on each end. The perimeter of the room is to be a 200-meter running track. Find the dimensions that will produce a maximum area of the rectangular region.

Perimeter $200 = 2\pi r + 2l \qquad l = \frac{200 - 2\pi r}{2}$

$A = l \cdot (2r)$

$A = \frac{200 - 2\pi r}{2} \cdot 2r = 200r - 2\pi r^2$



3. A rectangular page is to contain 24 square inches of print. The margins at the top and bottom of the page are to be 1.5 inches, and the margins on the left and right are to be 1 inch. What should the dimensions of the page be so that the least amount of paper is used?

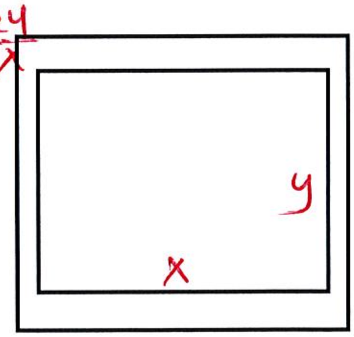
Print $= x \cdot y = 24 \qquad x = \frac{24}{y} \quad y = \frac{24}{x}$

$(x+2)(y+3) = \text{min}$

$(x+2)\left(\frac{24}{x} + 3\right) = \text{min}$

$24 + 3x + \frac{48}{x} + 6$

$3x + \frac{48}{x} + 30$



4. A movie screen on a wall is 20 feet high and 10 feet above the floor. At what distance x from the front of the room should position yourself so that the viewing angle, θ, of the movie screen is as large as possible?

$\tan(\theta + \alpha) = \frac{30}{x} \qquad \tan \alpha = \frac{10}{x}$

(We will come back to this).