12.6 Binomial Theorem

1. **Binomial Theorem:** If n is a nonnegative integer then

$$(x+y)^n = x^n + nx^{n-1}\bullet y + \ldots {}_nC_r x^{n-r}\bullet y^r$$

where ${}_nC_r = \dfrac{n!}{(n-r)!r!}$ if you are looking for the $\boxed{\text{fifth term then r = 4}}$

↑

$$(a+b)^n = 1a^nb^0 + \frac{n}{1}a^{n-1}b^1 + \frac{n(n-1)}{1\bullet 2}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{1\bullet 2\bullet 3}a^{n-3}b^3 + \ldots\ldots + 1a^0b^n$$

$$= \frac{n!}{n!\bullet 0!}a^nb^0 + \frac{n!}{(n-1)!\bullet 1!}a^{n-1}b^2 + \frac{n!}{(n-2)!\bullet 2!}a^{n-3}b^3 + \ldots + \frac{n!}{0!\bullet n!}$$

$$= \sum_{k=0}^{n}\frac{n!}{(n-k)!\bullet k!}a^{n-k}b^k$$

$\boxed{{}_nC_r \text{ is MATH} \to \text{PRB}}$

The key is knowing that $(a+b)^n$ gets expanded and that the 3rd term has b^2, 4th term has b^3, ..., etc.

This tells us r is one less than the sought out term.

a^(degree − r) and the multiplier (Pascal's) is ${}_nC_r$

2. Use the binomial theorem to expand each expression.
   a. $(x-5)^4$

$${}_4C_0\, x^4 5^0 - {}_4C_1\, x^3 5^1 + {}_4C_2\, x^2 5^2 - {}_4C_3\, x^1 5^3 + {}_4C_4\, x^0 5^4$$

↑ first term

3. Find the indicated term of each expression.

   a. Fourth term of $(x+2)^7$

   ↑ Tells me 2 must be cubed

$${}_7C_3 (x)^4 2^3 = 35\cdot x^4\cdot 8 = \boxed{270x^4}$$

   b. Sixth term of $(x-y)^9$

   ↑ Tells me y must be 5

$${}_9C_5 (x^4)(y)^5 = \boxed{-126x^4y^5}$$

   ↑ power; ↑ must be negative.

   c. Fifth term of $(2x+3y)^9$

$${}_9C_4 (2x)^5 (3y)^4$$

$$126\cdot 32\cdot x^5\cdot 81\cdot y^4 = \boxed{326{,}592x^5y^4}$$

   d. Third term of $(a-2\sqrt{3})^6$

   odd term ↑ +

$${}_6C_2\, a^4 (2\sqrt{3})^2 =$$

$$15a^4\cdot 12 = \boxed{180a^4}$$

   e. Find the middle term of $(4x^2-9y)^{12}$

   7th term

$${}_{12}C_6 (4x^2)^6 (9y)^6 = 924\bullet 4096\bullet x^{12}\bullet 531441y^6 = \text{HUGE}$$

$$2.011346878\times 10^{12}\, x^{12}y^6$$

4. Find the term containing y^8 in the expansion of $(2x+3y^2)^9$

power to power $\rightarrow (3y^2)^4$ so ${}_9C_4 (2x)^5 (3y^2)^4 =$

$$126\cdot 32\cdot x^5\cdot 81\cdot y^8 = \boxed{326{,}592x^5y^8}$$