9. Growth and Decay Formula:

A general rule for growth and decay is $y = ne^{kt}$ y = ne^{kt}.

10. The half-life of Phosphorus-33 is about 25 days. When will a 20-gram sample of P-33 be reduced to 8 grams?

$8 = 20\left(\frac{1}{2}\right)^{t/25}$

$8/20 = \left(\frac{1}{2}\right)^{t/25}$

$\log .4 = t/25 \log \frac{1}{2}$

$t/25 = \frac{\log .4}{\log 1/2}$

$t = \frac{25 \log .4}{\log 1/2}$

OR

$.4 = \left(\frac{1}{2}\right)^{t/25}$

$\log_{1/2} .4 = t/25 \cancel{\log_{1/2} 1/2}$

$25 \log_{1/2} .4 = t$

11. A major highway was constructed five years ago to accommodate a population of up to 40,000 commuters. It was estimated that the commuter population at that time was about 25,000. Today, there are 31,000 cars commuting on the highway each day.
   a. If the commuter population continues to grow at this rate, when will the highway need to be upgraded again?

$31{,}000 = 25000 e^{5K}$

$31/25 = e^{5K}$

$\ln 31/25 = 5K \cancel{\ln e}$

$K = \frac{\ln 31/25}{5} \approx .043$

$40{,}000 = 25{,}000 e^{.043t}$

$\ln 40/25 = \ln e^{.043t}$

$\ln 40/25 = .043t \cancel{\ln e}$

$t \approx 10.9$ years

6 years from now.

   b. How many commuters will the upgraded highway have to accommodate to meet the demand for an additional 10 years?

$= 40{,}000 e^{.043(10)}$

↑ upgrade when reach 40,000

$= 61{,}490.3$

12. If two languages have evolved separately from a common ancestral language, the number of years since the split, n(r), is given by the formula $n(r) = -5000 \ln r$, where r is the percentage of the words from the ancestral language that are common to both languages. Suppose two languages split off from a common language 1000 years ago. What portion of the words from the ancestral language would you expect to find in each of them today?

$1000 = -5000 \ln r$

$-1/5 = \ln r$

$e^{-1/5} = r$

$.8187 = r$

DAY 5 HW

11.7 Book pg. 643 #31-36

31. Radium 226, which is used for cancer treatment and as an ingredient in fluorescent paint, decomposes radioactively. Its half-life is 1800 years. Find the constant k you would use in the decay formula for radium. Use 1 gram as the original amount.

Skip.

32. Mr. Cuthbert invested a sum of money in a certificate of deposit that pays 8% interest compounded continuously. Recall that the formula for the amount in an account earning interest compounded continuously is $A = Pe^{rt}$. If Mr. Cuthbert made the investment on January 1, 1986 and the account is worth $10,000 on January 1, 2005, what was the original amount in the account?

$10{,}000 = Pe^{19(.08)}$

$P = 2{,}187.12$

3. DDT is an insecticide that has been used by farmers. It decays slowly and is sometimes absorbed by plants that animals and humans eat. DDT absorbed in the mud at the bottom of a lake is degraded into harmless products by bacterial action. Experimental data shows that 10% of the initial amount is eliminated in 5 years.

a. Find the value of k in the decay formula. $.9 = e^{5k}$ $\ln .9 = 5k \ln e$ $k = -.021$

b. How much of the original amount of DDT is left after 10 years? $e^{-.021(10)} = .81$ or 81%

c. The U. S. Environmental Protection Agency banned almost all use of DDT in the U. S. in 1972. If none has been used near the late since then, in what year will the concentration of DDT fall below 25%?

$.25 = e^{-.021 t}$ $t = \frac{\ln .25}{-.021} \approx 66$ years.

$\ln .25 = -.021t \ln e$

34. Sales of a product under relatively stable market conditions tend to decline at a constant annual rate in the absence of promotional activities. This sales decline can be expressed by the exponential function of the form $s = s_0 e^{-at}$, where s is the sales time at time t, t is time in years, s_0 is the sales at time t = 0, and a is the sales decay constant. Suppose sales of On-Time Watches were 45,000 the first year and 37,000 the second year.

a. Find the value of a in the equation for this sales decline. $37000 = 45{,}000 e^{-a(2)}$ $a \approx 9.78\%$

b. Find the projected sales for three years from now. $37{,}000 e^{-3(.0978)} \approx 27{,}585.80$

c. If the trend continues, when would you expect sales to be 15,000 units?

$15{,}000 = 37{,}000 e^{-.0978t}$

$15/37 = e^{-.0978t}$ $t \approx 9.23$ years.

35. Mike Kallenberg deposited some money in a bank account that earns 5.6% interest compounded continuously.

a. How long would it take to double the amount of money in Mr. Kallenberg's account? $2 = e^{.056t}$ $t \approx 12.38$ years.

b. The Rule of 72 says that if you divide 72 by the interest rate of an account that compounds interest continuously, the result is the approximate number of years that it will take for the money in the account to double. Do you think that the rule of 72 is accurate? Explain.

$72/5.6 \approx 12.86$ years.

36. The atmospheric pressure varies with the altitude above the surface of the Earth. Meteorologists have determined that for altitudes for up to 10 km, the pressure p in millimeters of mercury is given by $p = 760e^{-0.125a}$, where a is the altitude in kilometers.

a. What is the atmospheric pressure at an altitude of 3.3 km? $p = 760 e^{-.125(3.3)}$ $p = 503.11$

b. At what altitude will the atmospheric pressure be 450 m of mercury?

$.45 \text{ km} = 760 e^{-.125a}$

$59.45 = a$

Application Notes:

1. **Compound Interest:**

The initial principal P is invested at r percent (expressed in decimal form).
The accumulated amount after t years is given by

$A = P\left(1+\frac{r}{n}\right)^{nt}$ when interest is compounded n times per year

$A = Pe^{rt}$ when interest in compounded continually

Ex: If \$750 is invested at 8% annual interest that is compounded monthly, when will the investment be worth \$1600?

$1600 = 750\left(1+\frac{.08}{12}\right)^{12t}$

$\frac{32}{15} = (1+.00666)^{12t}$

$\log\left(\frac{32}{15}\right) = 12t\log(1.006666)$

$\log\left(\frac{32}{15}\right)/12\log(1.00666) = t$ $\quad t \approx 9.5$ years

Ex: A piece of office equipment worth \$8000 depreciates at 7.5% per year for the first ten years. At this rate, when will the piece of equipment be worth \$5000?

$5000 = 8000(1-.075)^{t}$

$\frac{5}{8} = (.925)^{t}$

$\log(5/8) = t\log .925$

$\log(5/8)/\log .925 = t$ $\quad t \approx 6.03$ years

Ex: If \$100 is invested at 12% annual interest that is compounded continuously, when will the investment be worth \$250?

$250 = 100e^{.12t}$

$2.5 = e^{.12t}$

$\ln 2.5 = .12t \cancel{\ln e}$

$\frac{\ln 2.5}{.12} = t$ $\quad t \approx 7.64$ years.

2. **Half Life:** $A = P\left(\frac{1}{2}\right)^{\frac{time}{half\ life\ rate}}$

Ex: Technetium-99 has a half-life of 6 hours. Suppose a lab has 80 mg of technetium-99. How much technetium-99 is left after 24 hours?

$A = 80\left(\frac{1}{2}\right)^{24/6}$ $\quad A = 5$ mg

Ex: Phosphorus-32 is a radioactive substance with a half-life of 14.3 days. How long would it take to reduce a 100 gram sample of P-32 to 15 grams?

$15 = 100\left(\frac{1}{2}\right)^{\frac{t}{14.3}}$ $\quad t/14.3 = \frac{\log .15}{\log 1/2}$

$.15 = \left(\frac{1}{2}\right)^{t/14.3}$ $\quad t = \frac{14.3\log .15}{\log 1/2}$

$\log .15 = (t/14.3)\log 1/2$

Ex: Cesium-137 has a half-life of 30 years. Suppose a lab stored a 30-mCi sample in 1973. How much of the sample will be left in 2003? In 2063?

$A = 30\left(\frac{1}{2}\right)^{30/30}$ $\quad A = 30\left(\frac{1}{2}\right)^{90/30}$

$A = 15$ $\quad A = 30/8 \approx 3.75$ mCi

3. Under ideal conditions, the population of a certain bacterial colony will double in 45 minutes. How much time will it take for the population to increase by 5 fold?

$50 = 10(2)^{t/45}$ $\quad \log 5 = t/45\log 2$

$5 = 2^{t/45}$ $\quad \frac{45\log 5}{\log 2} = t$

$t \approx 104.5$ mins.

4. A population of insects is growing in such a way that the number in the population t days from now is given by the formula $A = 4000e^{0.02t}$. How large will the population be in one week?

$A = 4000e^{.02(7)}$

$A = 4,602$ (round to next insect).

5. In 1990, the population of Washington, D.C., was about 604,000 people. Since then the population has decreased about 1.8% per year.

   a. Write an equation to model the population of Washington, D.C., since 1990.

   $A = 604,000e^{-.018t}$

   b. Suppose the current trend in population change continues. Predict the population of Washington, D.C., in 2010.

   $A = 604,000\, e^{-.018(20)}$.

   $A = 421,397$ people.

6. Suppose your community has 4512 students this year. The student population is growing 2.5% each year.

   a. Write an equation to model the student population. $A = 4512e^{.025t}$

   b. What will the student population be in 3 years?

   $A = 4512\, e^{(.025)(3)}$

   $A = 4,864$ (round to next person)

7. Logistic Growth Models: $y = \dfrac{a}{1 + be^{-rx}}$

Ex: The function $f(t) = \dfrac{30,000}{1 + 20e^{-1.5t}}$ describes the number of people, $f(t)$ who have become ill with influenza t weeks after its initial outbreak in a town with 30,000 inhabitants.

   a. How many people became ill with the flu when the epidemic began?

   $t = 0$ $\quad \dfrac{30,000}{21} \approx 1,429$

   b. When will the 5000th person be infected at the current rate of the outbreak?

   $5000 = \dfrac{30,000}{1 + 20e^{-1.5x}}$

   $5000(1 + 20e^{-1.5x}) = 30,000$

   $1 + 20e^{-1.5x} = 6$

   $e^{-1.5x} = 1/4$

   $x = .924$ weeks.

Worksheet; page 617 #25-28, 31-33; page 640 #42; page 643 #31-35, Richter Scale Worksheet, Application Worksheet