12.5 Sigma Notation & The nth Term

I. Write each of the following in expanded form and find the sum.

1. $\sum_{n=4}^{7}(3^n+1)$

$82+244+730+2{,}188$

$=3{,}244$

2. $\sum_{n=1}^{\infty}2\left(\frac{1}{3}\right)^n$

$\frac{2}{3}+\frac{2}{9}+\frac{2}{27}\ldots \quad S_n=\frac{\frac{2}{3}}{1-\frac{1}{3}}=1$

3. Express $16+19+22+25+28$ using sigma notation.

$d=3$

$a_n=3n+13$

\# of terms $\sum_{n=1}^{5}(3n+13)$ don't forget!

4. Express $25-6.25+1.5625-.390625$ using sigma notation.

$r=\frac{-6.25}{25}=\frac{-1}{4}$

$\sum_{n=1}^{4}25\left(\frac{-1}{4}\right)^{n-1}$

5. Not all sequences are arithmetic or geometric;
Some important sequences are generated by products of consecutive integers

Factorial: The product $n(n-1)(n-2)\ldots1$ **is called** <u>***n*** **factorial**</u> **and is symbolized by n!**

6. **By Definition** $0!=1$

7. Evaluate each expression: ! (MATH → PRB)

a. $5!$

$5\cdot4\cdot3\cdot2\cdot1=$ $\boxed{120}$

b. $12!$

$\boxed{479{,}001{,}600}$

c. $\frac{10!}{3!\bullet 6!}$

$\frac{10\cdot\overset{3}{\cancel{9}}\cdot\cancel{8}\cdot7\cdot\cancel{6}\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}{\cancel{3}\cdot\cancel{2}\cdot1\ \ \cancel{6}\cdot\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot\cancel{1}}=$ $\boxed{840}$

d. $\frac{3!}{6!-3!}$

$=\frac{3!}{3!}\left(\frac{1}{6\cdot5\cdot4-1}\right)$ OR

$\frac{3\cdot2\cdot1}{6\cdot5\cdot4\cdot3\cdot2\cdot1-3\cdot2\cdot1}=\frac{6}{714}$

$\boxed{\frac{1}{119}}$

e. $\frac{n!}{(n+1)!}$

$\frac{\cancel{n}(\cancel{n-1})(\cancel{n-2})(\cancel{n-3})\ldots}{(n+1)(\cancel{n})(\cancel{n-1})(\cancel{n-2})\ldots}$

$=\boxed{\frac{1}{n+1}}$

f. $\frac{9!}{0!}=9!$

$\boxed{362{,}880}$

g. $\frac{3!+6!}{6!-3!}$

$\frac{3\cdot2\cdot1+6\cdot5\cdot4\cdot3\cdot2\cdot1}{6\cdot5\cdot4\cdot3\cdot2\cdot1-3\cdot2\cdot1}$

$=\frac{726}{714}=\boxed{\frac{121}{119}}$

h. $\frac{(x+1)!(x-2)!}{(x-4)!(x-1)!}$

$\frac{(x+1)(x)\cancel{(x-1)}(x-2)(x-3)\cancel{(x-4)}\ldots(x-2)(x-3)(x-4)}{\cancel{(x-4)}(x-5)\ldots\ \cancel{(x-1)}(x-2)(x-3)\ldots}$

$\boxed{(x+1)(x)(x-2)(x-3)}$

12.6 Pascal's Triangle

$(x+y)^0 \to 1$

$(x+y)^1 \quad 1 \quad 1$

$(x+y)^2 \quad 1 \quad 2 \quad 1$

$(x+y)^3 \quad 1 \quad 3 \quad 3 \quad 1$

$(x+y)^4 \quad 1 \quad 4 \quad 6 \quad 4 \quad 1$

$1 \quad 5 \quad 10 \quad 10 \quad 5 \quad 1$

$1 \quad 6 \quad 15 \quad 20 \quad 15 \quad 6 \quad 1$

1. Use Pascal's Triangle to expand each binomial.

$\boxed{\text{first terms power decreases as 2nd terms power increases}}$

a. $(x+y)^3$

Instead of $(x+y)(x+y)(x+y)$ and FOILING!

$1x^3y^0 \quad 3x^2y^1 \quad 3x^1y^2 \quad 1x^0y^3$

$= \boxed{x^3 + 3x^2y + 3xy^2 + y^3}$

b. $(x-3)^5$

ignore − this will make every other sign change!

$1x^5 3^0 \quad 5x^4 3^1 \quad 10x^3 3^2 \quad 10x^2 3^3 \quad 5x^1 3^4 \quad 1x^0 3^5$

$x^5 - 15x^4 + 90x^3 - 270x^2 + 405x - 243$

c. $(x+2y)^6$

both get squared

$1x^6(2y)^0 \quad 6x^5(2y)^1 \quad 15x^4(2y)^2 \quad 20x^3(2y)^3 \quad 15x^2(2y)^4 \quad 6x^1(2y)^5 \quad 1x^0(2y)^6$

$x^6 + 12x^5 + 60x^4y^2 + 160x^3y^3 + 240x^2y^4 + 192xy^5 + 64y^6$

d. $(a-5)^3$

$1a^3 5^0 \quad -3a^2 5^1 \quad 3a^1 5^2 \quad -1a^0 5^3$

$a^3 - 15a^2 + 75a - 125$

e. $(3x-2y)^5$

$(-5)(81) \quad (2)$

$1(3x)^5(2y)^0 \quad -5(3x)^4(2y)^1 \quad 10(3x)^3(2y)^2 \quad -10(3x)^2(2y)^3 \quad 5(3x)^1(2y)^4 \quad -1(3x)^0(2y)^5$

$243x^5 - 810x^4y + 1080x^3y^2 - 720x^2y^3 + 240xy^4 - 32y^5$

Homework page 688 # 23-33 odd, 35-38, 41, 42; Worksheet Day 6 on Factorials and Pascal's