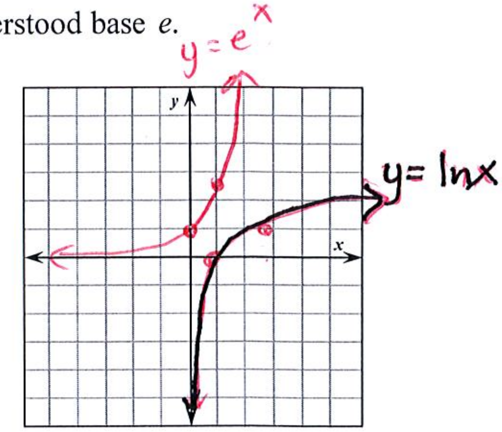
11.7 Natural Logarithms

1. **Natural Logarithms:**
   The natural log function is the inverse of $y = e^x$. Denoted $\ln x$, understood base e.
   All properties of logarithms also hold for natural logarithms.



2. Graph $y = e^x$ *and* $y = \ln x$ $y = \log_e x$

3. Write in logarithmic form.
   a. $e^{-x} = 5$ b. $e^2 = 6x$
   $\ln 5 = -x$ or $\log_e 5 = -x$

4. Write in exponential form.
   a. $\ln e = 1$ b. $\ln 5.2 = x$
   $e^1 = e$ $e^x = 5.2$

5. Evaluate each:
   $\cancel{e^{\log_e y}}$ $e^{?} = e^{-4x}$
   a. $\cancel{e^{\ln .2}} = .2$ b. $e^{\ln y} = y$ c. $\ln e^{-4x} = -4x$ $\log_e e^{-4x}$ d. $\ln e^{45}$ 45

6. Use the properties of logarithms to write the expression in expanded form. (all variables are positive.)
   a. $\ln 4x^2$ b. $\ln\left(\frac{e^2}{5}\right)$
   $\ln 4 + 2\ln x$ $2\ln e - \ln 5$
   $2 - \ln 5$

7. Write the expression as the logarithm of a single quantity.
   a. $\ln x + \ln 2$ b. $\frac{1}{2}\ln x + 3\ln y$ c. $4\ln x + 7\ln x - 3\ln x$
   $\ln 2x$ $\cancel{\ln \frac{\sqrt{x}}{y^3}}$ $\ln x^4 + \ln x^7 - \ln x^3$
   $\ln y^3\sqrt{x}$ $\ln x^4 \cdot x^7 - \ln x^3$
   $\ln x^{11} - \ln x^3 = \ln x^8$
   $\boxed{8\ln x}$

8. Solve each.
   a. $\ln N = 4.987$ b. $\ln N = 0.7831$ c. $e^x = 46$ d. $e^{4k} = 18$
   $e^{4.987} = N$ $e^{.7831} = N$ $\ln e^x = \ln 46$
   $\boxed{146.4963 = N}$ $x \ln \cancel{e} = \ln 46$ $4k \ln \cancel{e} = \ln 18$
   $\ln 19.8 = .083t$ $k = \frac{\ln 18}{4}$

   e. $3e^{.035x} = 519$ f. $\ln 19.8 = \ln e^{0.083t}$ g. $\ln 6.2 = \ln e^{0.55t}$
   $e^{.035x} = 173$ $19.8 = e^{.083t}$ $6.2 = e^{.55t}$
   $.035x \ln \cancel{e} = \ln 173$ $\frac{\ln 19.8}{.083} = t$ $\frac{\ln 6.2}{.55} = t$
   $x = \ln 173 / .035$

9. Growth and Decay Formula:

A general rule for growth and decay is $y = ne^{kt}$.

10. The half-life of Phosphorus-33 is about 25 days. When will a 20-gram sample of P-33 be reduced to 8 grams?

$8 = 20\left(\frac{1}{2}\right)^{\frac{\text{time}}{\text{½ liferate}}}$ $\frac{2}{5} = \left(\frac{1}{2}\right)^{\frac{t}{25}}$

$.4 = \left(\frac{1}{2}\right)^{t/25}$ $\log .4 = t/25 \log(.5)$

$\frac{25 \log .4}{\log .5} = \cancel{t/25}$ $\boxed{t \approx 33 \text{ days}}$

11. A major highway was constructed five years ago to accommodate a population of up to 40,000 commuters. It was estimated that the commuter population at that time was about 25,000. Today, there are 31,000 cars commuting on the highway each day.

   a. If the commuter population continues to grow at this rate, when will the highway need to be upgraded again?

$\frac{31{,}000 - 25{,}000}{25{,}000}$

$.24 / 5$ A.P.R.

$A = 25{,}000\, e^{.048t}$

$40{,}000 = 25{,}000\, e^{.048t}$

$\frac{40}{25} = e^{.048t}$ $\ln(40/25) = .048t \ln e$

$\boxed{9.8 \text{ years} = t}$

   b. How many commuters will the upgraded highway have to accommodate to meet the demand for an additional 10 years?

$A = 25{,}000\, e^{.048(19.8)}$

$A = 64{,}669$ cars.

12. If two languages have evolved separately from a common ancestral language, the number of years since the split, n(r), is given by the formula $n(r) = -5000 \ln r$, where r is the percentage of the words from the ancestral language that are common to both languages. Suppose two languages split off from a common language 1000 years ago. What portion of the words from the ancestral language would you expect to find in each of them today?

$1000 = -5000 \ln r$

$-1/5 = \ln r$

$e^{-1/5} = r$

$.8187 = r$

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