12.3 and 12.4 Infinite Sequences & Series

1. **Infinite Sequence:** A sequence that never ends.

2. The sequence $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots, \frac{1}{n}$ has infinitely many terms; as n increases the value of the terms decrease and get closer and closer to 0. Think about the graph of this function.

   0 is called the Limit of the terms in this sequence and can be expressed as follows:

   $\lim_{n \to \infty} \frac{1}{n} = 0$ *"the limit of $\frac{1}{n}$, as n approaches infinity equals zero".*

3. Rules for Finding Limits (**Same as horizontal asymptote rules**):

   a. If the largest exponents are the same in the numerator and denominator, the limit is the ratio of the coefficients of the terms containing the largest exponent.

   b. If largest exponent is in the numerator, there is no limit.

   c. If the largest exponent is in the denominator, the limit is 0.

4. Find each limit:

   a. $\lim_{n \to \infty} \frac{1-2n}{5n}$

   $\lim_{n \to \infty} \frac{-2n+1}{5n} = \frac{-2}{5}$

   b. $\lim_{n \to \infty} \frac{4n^2-6}{3n-1}$

   $\lim_{n \to \infty} \frac{4n^2-6}{3n-1} = \text{DNE}$

   c. $\lim_{n \to \infty} \frac{2n^2-3n+4}{3n^2+1}$

   $\lim_{n \to \infty} \frac{2n^2-3n+4}{3n^2+1} = \frac{2}{3}$

   d. $\lim_{n \to \infty} \frac{n^2+4}{n^3}$

   $\lim_{n \to \infty} \frac{n^2+4}{n^3} = 0$

   $\lim_{n \to \infty} \frac{n^2}{n^3} + \lim_{n \to \infty} \frac{4}{n^3}$

   *e. $\lim_{n \to \infty} \frac{1}{5^n}$

   Remember this is $\lim_{n \to \infty} \left(\frac{1}{5}\right)^n$ so $\lim_{n \to \infty} \left(\frac{1}{5}\right)^n = 0$

   *f. $\lim_{n \to \infty} (2)^n$

   $\lim_{n \to \infty} 2^n = \text{DNE}$

5. **Infinite Series:** An infinite series is the indicated sum of the terms of an infinite sequence

6. **Sum of an Infinite Series:** The sum of an infinite geometric series for which $|r| < 1$ is

   given by $\boxed{S_n = \frac{a_1}{1-r}}$

We knew $S_n = a_1\left(\frac{1-r^n}{1-r}\right)$

$= a_1\left(\frac{1-(\quad)^n}{1-r}\right)$

now if r is a fraction the limit goes to zero.

7. Find the sum of each infinite series, or state that the sum does not exist.

a. $\frac{1}{20}+\frac{1}{40}+\frac{1}{80}+\ldots$

$r = \frac{1}{2}$

$S_n = \frac{\frac{1}{20}}{1-\frac{1}{2}} = \frac{1}{20} \div \frac{1}{2} = \boxed{\frac{1}{10}}$

b. $2\sqrt{2}+8+16\sqrt{2}\ \ldots$

$r = 2\sqrt{2}$

Since $|r|>1$ the sum DNE

c. $2+\sqrt{2}+1+\ \ldots$

$r = \frac{\sqrt{2}}{2}$

$S_n = \frac{2}{1-\frac{\sqrt{2}}{2}} = \frac{2}{\frac{2-\sqrt{2}}{2}} = 2 \div \frac{2-\sqrt{2}}{2}$

$2 \cdot \frac{2}{2-\sqrt{2}} = \frac{4}{2-\sqrt{2}}\left[\frac{2+\sqrt{2}}{2+\sqrt{2}}\right] = \frac{8+4\sqrt{2}}{2}$

$= 4+2\sqrt{2}$

8. **Convergent Series:** Has a sum or limit ($|r|<1$)

9. **Divergent Series:** Does not have a sum or limit, ($|r|\geq 1$) and it diverges

10. Determine whether each series is arithmetic or geometric, then determine if it is convergent or divergent.

a. $\frac{2}{3}+\frac{1}{3}+\frac{1}{6}+\ \ldots$

$r = \frac{1}{2}$ geometric

convergent

$S_n = \frac{\frac{2}{3}}{1-\frac{1}{2}} = \frac{2}{3}\cdot\frac{2}{1} = \frac{4}{3}$

b. $-4-2-0+2+\ \ldots$

$d = 2$ arithmetic

divergent

c. $1+3+9+27+\ \ldots$

$r = 3$ geometric

divergent

d. $\sum_{k=1}^{\infty} 2^k$

$2+4+8+16, \ldots$

$r = 2$ geometric

divergent

f. $\sum_{n=1}^{\infty} 2(.4)^k$

$.8 + .32 + .128$

$r = .4$

convergent

$S_n = \frac{.8}{1-.4} = \frac{4}{3}$

11. **Repeating Decimals as a Fraction:**
You can use what you know about infinite series to write repeating decimals as fractions - - you must write the repeating decimal as an infinite geometric series

12. Write each repeating decimal as a fraction.

a. 0.454545...

$\frac{45}{100}+\frac{45}{10{,}000}+\frac{45}{1{,}000{,}000}\ldots$

$r = \frac{1}{100}$

$\frac{a_1}{1-r} = \frac{\frac{45}{100}}{1-\frac{1}{100}} = \frac{45}{100} \div \frac{99}{100} = \boxed{\frac{45}{99}}$

$= \frac{5}{11}$

b. 0.888...

$\frac{8}{10}+\frac{8}{100}+\frac{8}{1000}$

$r = \frac{1}{10}$

$\frac{\frac{8}{10}}{1-\frac{1}{10}} = \frac{8}{10} \div \frac{9}{10}$

$= \boxed{\frac{8}{9}}$

c. 7.259259...

$7 + \left[\frac{259}{1000} + \frac{259}{1{,}000{,}000}\right.$

$r = \frac{1}{1000}$

$7 + \frac{\frac{259}{1000}}{1-\frac{1}{1000}}$

$7 + \frac{259}{999}$

$= \frac{7252}{999} = \boxed{\frac{196}{27}}$

Homework: page 676 # 17-24; Worksheet Day 5