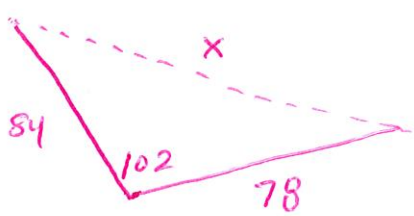
5.7 Law of Cosines

1. **Law of Cosines:**

Let triangle ABC be any triangle with a, b, c representing the measures of the sides opposite the angle with measurements A, B, and C respectively. Then the following is true.

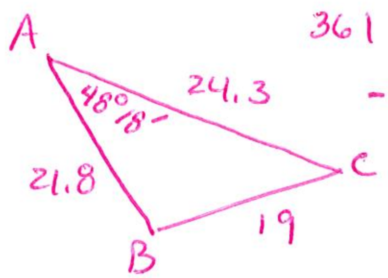
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

2. Suppose you want to fence a triangular lot as shown below. If two sides measure 84 feet and 78 feet and the angle between the two sides is $102°$, what is the length of the fence to the nearest foot?



$$x^2 = 84^2 + 78^2 - 2(84)(78)\cos 102^\circ$$
$$x^2 = 15864.47$$
$$\boxed{x = 126 \text{ ft}}$$

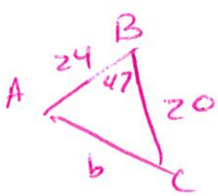
3. Solve triangle ABC if $a = 19. b = 24.3,$ and $c = 21.8$. Round angle measures to the nearest minute.



$$19^2 = 24.3^2 + 21.8^2 - [2(24.3)(21.8)\cos A]$$
$$361 = 590.49 + 475.24 - [1059.48\cos A]$$
$$-704.73 = -[1059.48\cos A]$$
$$\frac{704.73}{1059.48} = \cos A$$
$$\cos^{-1}(\ ) = A \quad \boxed{A = 48^\circ 18'}$$
$$\frac{\sin 48^\circ 18'}{19} = \frac{\sin B}{24.3}$$
$$\boxed{B = 72^\circ 44'} \quad \boxed{C = 58^\circ 58'}$$

4. Determine whether each triangle should be solved by beginning with the Law of Sines or the Law of Cosines. Then solve each triangle. Round measures of sides to the nearest tenth and measure of angles to the nearest degree.

a. $a = 20, c = 24, B = 47°$



$$b^2 = 24^2 + 20^2 - [2(24)(20)\cos 47]$$
$$b^2 = 321.28$$
$$\boxed{b = 17.9}$$
$$\frac{\sin 47}{17.9} = \frac{\sin A}{20}$$
$$\boxed{A = 54^\circ 48'} \quad \boxed{C = 78^\circ 12'}$$

b. $a = 345, b = 648, c = 442$

$$345^2 = 648^2 + 442^2 - [2(648)(442)\cos A]$$
$$\boxed{30^\circ = A}$$
$$\frac{\sin 30}{345} = \frac{\sin B}{648}$$
$$\boxed{B = 69^\circ 54' \quad C = 80^\circ 6'}$$

c. $A = 36°, a = 10, b = 19$ (Sines)

$\frac{\sin 36}{10} = \frac{\sin B}{19}$

no Solution

d. $A = 25°, B = 78°, a = 13.7$ (Sines)

$\frac{\sin 25}{13.7} = \frac{\sin 78}{b}$

$\boxed{b = 31.7}$ $\boxed{C = 77}$

$\frac{\sin 25}{13.7} = \frac{\sin 77}{c}$ $\boxed{c = 31.6}$

5.8 Area of Triangles

1. **Area of a Triangle with a Known Height:**

$$A = \frac{1}{2}bh$$

Formulas for finding area of Oblique Triangle:

$A = \frac{1}{2} ab \sin C$

$A = \frac{1}{2} bc \sin A$

$A = \frac{1}{2} ac \sin B$

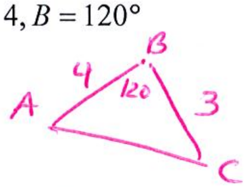
2. Find the area of each triangle described below. Round answers to the nearest tenth.

A. $a = 8.4, b = 10, C = 108°$

$A = \frac{1}{2} \cdot 8.4 \cdot 10 \cdot \sin 108°$

$\boxed{A = 39.9 \ u^2}$

B. $a = 3, c = 4, B = 120°$



$A = \frac{1}{2}(3)(4) \sin 120$

$\boxed{A = 5.2 \ u^2}$

C. $a = 14.2, A = 18°50'$, $B = 69°18'$

$c = 91°52'$

$\frac{\sin 18°50'}{14.2} = \frac{\sin 91°52'}{c}$

$c = 44$

$A = \frac{1}{2}(14.2)(44) \sin 69°18'$

$\boxed{A = 292.2 \ u^2}$

3. **Heron's Formula:**

If the measures of the sides of a triangle are a, b, c, then the area K, of the triangle is as follows:

$$K = \sqrt{S(s-a)(s-b)(s-c)}, \text{ where } s = \frac{1}{2}(a+b+c) \quad \textbf{(s = semiperimeter)}$$

4. Use Heron's formula to find the area of each triangle.

A. $a = 30, b = 50, c = 56$

$s = 136/2 = 68$

$A = \sqrt{68(38)(18)(12)} = \boxed{747.1 \ u^2}$

B. $a = 5, b = 12, c = 13$ $S = 15$

$K = \sqrt{15(15-5)(15-12)(15-13)}$

$\boxed{K = 30 \ u^2}$