

4. **Slant Asymptote:** They are exactly how they sound. \*Neither vertical or horizontal!! These occur only if the degree of the numerator is exactly one more than the degree of the denominator.

Need to do synthetic or long division to find it.

Ex.  $p(x) = \frac{x^2 - 2x + 1}{x + 3}$

$$\begin{array}{r} -3 \overline{) 1 \ -2 \ 1} \\ \underline{-3 \ 15} \\ 1 \ -5 \ 16 \end{array}$$

$y = x - 5$

Ex:  $w(x) = \frac{3x^2 + 4}{x + 2}$

$$\begin{array}{r} -2 \overline{) 3 \ 0 \ 4} \\ \underline{-6 \ 12} \\ 3 \ -6 \ 16 \end{array}$$

$y = 3x - 6$

Ex: Long Division Example

5. Determine the vertical and horizontal asymptotes of the function, (include slant, if one). Also list any holes, if any. Then state the domain.

a.  $f(x) = \frac{5x}{x - 4}$

Hole: N/A  
 Domain:  $(-\infty, 4) \cup (4, \infty)$   
 VA:  $x = 4$   
 HA:  $y = 5$   
 SA: N/A  
 x-intercept:  $(0, 0)$   
 y-intercept:  $(0, 0)$

b.  $f(x) = \frac{-2x + 1}{3x + 5}$

Hole: N/A  
 Domain:  $(-\infty, -5/3) \cup (-5/3, \infty)$   
 VA:  $x = -5/3$   
 HA:  $y = -2/3$   
 SA: N/A  
 x-intercept:  $(1/2, 0)$   
 y-intercept:  $(0, 1/5)$

c.  $f(x) = \frac{3}{x^2 + 4}$

Hole: N/A  
 Domain:  $(-\infty, \infty)$   
 VA: N/A  
 HA:  $y = 0$   
 SA: N/A  
 x-intercept: N/A  
 y-intercept:  $(0, 3/4)$

d.  $g(x) = \frac{x^2 - 1}{x^2 - 6x - 7} \frac{(x-1)(x+1)}{(x-7)(x+1)}$

Hole:  $(-1, 1/4)$   
 Domain:  $(-\infty, -1) \cup (-1, 7) \cup (7, \infty)$   
 VA:  $x = 7$   
 HA:  $y = 1$   
 SA: N/A  
 x-intercept:  $(1, 0)$   
 y-intercept:  $(0, 1/7)$

e.  $g(x) = \frac{x^2 - 6x + 9}{x^2 - x - 6} \frac{x-3}{x-3} \frac{x-3}{x+2}$

Hole:  $(3, 0)$   
 Domain:  $(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$   
 VA:  $x = -2$   
 HA:  $y = 1$   
 SA: N/A  
 x-intercept:  $(3, 0)$   
 y-intercept:  $(0, -3/2)$

f.  $f(x) = \frac{x^2 + 6x + 8}{x + 4} \frac{x+4}{x+4} \frac{x+2}{x+4}$

Hole:  $(-4, -2)$   
 Domain:  $(-\infty, -4) \cup (-4, \infty)$   
 VA: N/A  
 HA: N/A  
 SA: N/A  
 x-intercept:  $(-2, 0)$   
 y-intercept:  $(0, 2)$

g.  $p(x) = \frac{12x}{3x^2 + 1}$

Hole: N/A  
 Domain:  $(-\infty, \infty)$   
 VA: N/A  
 HA:  $y = 0$   
 SA: N/A  
 x-intercept:  $(0, 0)$   
 y-intercept:  $(0, 0)$

h.  $g(x) = \frac{2x^3}{x^2 + 1}$

Hole: N/A  
 Domain:  $(-\infty, \infty)$   
 VA: N/A  
 HA: N/A  
 SA:  $y = 2x$   
 x-intercept:  $(0, 0)$   
 y-intercept:  $(0, 0)$

i.  $g(x) = \frac{x^2 - 9}{x + 2}$

Hole: N/A  
 Domain:  $(-\infty, -2) \cup (-2, \infty)$   
 VA:  $x = -2$   
 HA: N/A  
 SA:  $y = x - 2$   
 x-intercept:  $(3, 0), (-3, 0)$   
 y-intercept:  $(0, -9/2)$