

**Worksheet Day 4: Geometric/Arithmetic Sequences and Series:**

Find the  $n$ th term of the Arithmetic Sequence having the given values of  $a$ ,  $d$ , and  $n$ .

1.  $a_1 = 6, d = \frac{2}{3}, n = 10$

$a_n = \frac{2}{3}n + \frac{10}{3} \quad a_{10} = 12$

2.  $a_1 = 25x, d = -3x, n = 33$

$a_n = -3xn + 28x \quad a_{33} = -71x$

Find the specified term of each arithmetic sequence:

3. The twentieth term of  $x - 11y, x - 9y, x - 7y, \dots$

$a_n = (0x + 2y)n + (x - 13y)$

4. Which term of  $115, 108, 101, \dots$  is 17?

$a_{20} = x + 27y$

$a_n = -7n + 122$

$17 = -7n + 122 \quad n = 15$

Insert the specified number of arithmetic means in each case:

5. Seven, between 26 and  $-30$

$26 \text{ --- } \text{ --- } \text{ --- } \text{ --- } \text{ --- } \text{ --- } \text{ --- } -30 \quad 8d = -56 \quad d = -7$

6. two, between  $2x + 5y$  and  $5x - 4y$

$2x + 5y \text{ --- } \text{ --- } 5x - 4y \quad d = x - 3y$

Find the missing terms:

7.  $t_4 = 13, t_6 = 7, t_1 =$

$t_n = -3n + 25 \quad t_1 = 22$

8.  $t_2 = -11, t_8 = 19, t_7 =$

$t_n = 5n - 21 \quad t_7 = 14$

Solve.

9. A rocket fired vertically traveled 6 meters during the first second and (as long as its engines provided thrust) traveled 34 meters farther during each following second than in the one before. How far did it travel during the 12<sup>th</sup> second?

$\underline{6} \quad \underline{40} \quad \underline{74} \quad a_n = 34n - 28$

$a_{12} = 380 \text{ meters}$

10. If an employee hired at \$6850 a year is guaranteed annual salary increases of \$450, in which year of his employment will he first earn at least \$10,000 a year?

$a_n = 450n + 6400$

$10,000 = 450n + 6400 \quad n = 8 \text{ years}$

11. A pile of bricks has 53 bricks in the first row, 51 in the second row, 49 in the third row, and so on, and 1 brick in the top row. How many bricks are in the 14<sup>th</sup> row?

$a_n = -2n + 55$

$a_{14} = -2(14) + 55 \quad 27 \text{ bricks}$

Use the given data to find the sum of the arithmetic series.

12.  $a_1 = -53, n = 45, d = 3$

$a_n = 3n - 56$

$a_{45} = 79$

$S_{45} = \frac{45}{2}(-53 + 79) = 585$

13.  $a_1 = 7, a_n = 127, d = 2$

$a_n = 2n + 5$

$a_{127} = 259$

$S_{127} = \frac{127}{2}(7 + 259) =$

$127 = 2n + 5$

$61 = n$

$S_{127} = 4,087$

Solve:

14. A lecture hall has 20 seats in the front row and two seats more in each following row than in the preceding one. If there are 15 rows, what is the seating capacity of the hall?

$a_n = 2n + 18$

$S_{15} = \left(\frac{15}{2}\right)(20 + 48)$

$a_{15} = 2(15) + 18 = 48$

$= 510 \text{ seats}$

15. Roger paid off a debt to his father in 8 months by paying \$60 the 1<sup>st</sup> month, \$55 the second, \$50 the third, and so on. How much was the original debt?

$a_n = -5n + 65$

$S_8 = \frac{8}{2}(60 + 25)$

$a_8 = -5(8) + 65$

$S_8 = \$340$

Write the next three terms of the given geometric sequence, then write an expression for the nth term.

16. 3, 6, 12, .....  $24, 48, 96$

17.  $\frac{9}{2}, -\frac{3}{2}, \frac{1}{2}$   $-\frac{1}{6}, \frac{1}{18}, -\frac{1}{54}$

Find the specified term of the geometric sequence described.

18. tenth term if  $a_1 = \frac{3}{32}$  and  $r = -2$   $a_{10} = \frac{3}{32} (-2)^9 = -48$

19. Eighth term of 24, 12, 6, ...  $a_8 = 24 (\frac{1}{2})^7 = \frac{3}{16}$

Insert the given number of positive geometric means and write the resulting geometric sequence.

20. Two, between 2 and 54  $2 \text{ --- } 54$   $r=3$   $a_n = 2(3)^{n-1}$

21. Four, between 48 and  $\frac{3}{2}$   $48 \text{ --- } \frac{3}{2}$   $r = \frac{1}{2}$   $a_n = 48(\frac{1}{2})^{n-1}$

Solve:

22. Assuming no duplication of ancestors, how many great, great, great, grandparents did you have?

$4 \text{ --- } 4$   $a_4 = 4(2)^3$

23. A piece of real estate bought 5 years ago for \$25,600 increased in value 25% each year since then. What is it worth now?

$a_6 = 25,600 (1.25)^5$   $\frac{25,600}{a_1} \frac{a_2}{a_2} \frac{a_3}{a_3} \frac{a_4}{a_4} \frac{a_5}{a_5}$

24. A car traveled 32 meters in the first second after the brakes were applied and in each second after that traveled half as far as it had in the second before. How far did the car travel in the ten seconds after the brakes were applied?

$a_{10} = 32(\frac{1}{2})^9$   
 $S_{10} = 32 \left( \frac{1 - (\frac{1}{2})^{10}}{1 - \frac{1}{2}} \right) = 63,9375$

Find the sum of the geometric series described.

25.  $a_1=3, r=2, n=6$   $S_6 = 3 \left( \frac{1 - 2^6}{1 - 2} \right) = 189$

26.  $a_1=8, r=1.5, a_n=40.5$   $a_n = a_1 r^{n-1}$   $40.5 = 8(1.5)^{n-1}$   $n=5$   $S_n = 8 \left( \frac{1 - 1.5^5}{1 - 1.5} \right) = 105.5$

Of the five quantities  $a, a_n, r, n,$  and  $S_n$ , three are given. Find the other two. Some exercises have two sets of answers.

27.  $n=8, r=2, S_n=765$   $765 = a_1 \left( \frac{1 - 2^8}{1 - 2} \right)$   $a_1 = 3$   $a_n = 3(2)^{8-1}$   
 $a_8 = 384$

28.  $a_1=2, n=3, S_n=42$

$42 = 2 \left( \frac{1 - r^3}{1 - r} \right)$   $r \neq 1$   
 $21(1-r) = 1 - r^3$   $r=4$   $r=-5$   
 $r^3 - 21r + 20 = 0$   $a_3 = 2(4)^2$   $a_3 = 2(-5)^2$   
 $a_3 = 32$   $a_3 = 50$