4.3 Remainder and Factor Theorem and 4.4 Descartes Rule Notes

1. **Synthetic Division:**

a. $(x^2-5x-12)\div(x-3)$

$$\begin{array}{r|rrr} 3 & 1 & -5 & -12 \\ & & 3 & -6 \\ \hline & 1 & -2 & -18 \end{array}$$

$(3,-18)$

$x-2+\frac{-18}{x-3}$

b. $(3x^2+4x-12)\div(x-5)$

$$\begin{array}{r|rrr} 5 & 3 & 4 & -12 \\ & & 15 & 95 \\ \hline & 3 & 19 & 83 \end{array}$$

$3x+19+\frac{83}{x-5}$

2. **The Remainder Theorem:**

If a polynomial $P(x)$ is divided by $x-r$, the remainder is a constant, $P(r)$, and $P(x)=(x-r)\cdot Q(x)+P(r)$ where $Q(x)$ is a polynomial with degree one less than the degree of P(x).

3. Let $P(x)=2x^3+x^2-4x+3$ Show that $P(-1)$ is the remainder when $P(x)$ is divided by x + 1.

$$\begin{array}{r|rrrr} -1 & 2 & 1 & -4 & 3 \\ & & -2 & 1 & 3 \\ \hline & 2 & -1 & -3 & 6 \end{array}$$

$P(-1)=2(-1)^3+(-1)^2-4(-1)+3$

$=6$

4. **The Factor Theorem:**

The binomial x – r is a factor of the polynomial $P(x)$ if and only if $P(r)=0$.

5. Let $P(x)=x^3-x^2-5x-3$ Determine if x – 3 is a factor of $P(x)$.

$$\begin{array}{r|rrrr} 3 & 1 & -1 & -5 & -3 \\ & & 3 & 6 & 3 \\ \hline & 1 & 2 & 1 & 0 \end{array}$$

yes, because the remainder = 0.

6. Find the value of k so that each remainder is zero.

a. $(2x^3+kx^2+7x-3)\div(x-3)$

①
$$\begin{array}{r|rrrr} 3 & 2 & k & 7 & -3 \\ & & 6 & 3k+18 & 9k+75 \\ \hline & 2 & k+6 & 3k+25 & 0 \end{array}$$

$9k+75=3$ $\boxed{k=-8}$

②
$$\begin{array}{r|rrrr} 3 & 2 & k & 7 & -3 \\ & \downarrow & 6 & -6 & 3 \\ \hline & 2 & -2 & 1 & 0 \end{array}$$

$k+6=-2$

b. $(x^3+9x^2+kx-12)\div(x+4)$

$$\begin{array}{r|rrrr} -4 & 1 & 9 & k & -12 \\ & & -4 & -20 & \\ \hline & 1 & 5 & k-20 & \end{array}$$

$k-20=-3$

$k=17$

7. Show that 1 is a zero of multiplicity of 2 of the polynomial function $P(x) = x^4 - 2x^2 + 1$ and express $P(x)$ as a product of linear factors.

$P(x) = (x-1)^2(x+1)^2$

$$\begin{array}{r|rrrrr} 1 & 1 & 0 & -2 & 0 & 1 \\ & & 1 & 1 & -1 & -1 \\ \hline 1 & 1 & 1 & -1 & -1 & \boxed{0} \\ & & 1 & 2 & 1 & \\ \hline & 1 & 2 & 1 & \boxed{0} & \end{array}$$

This is now a Quadratic → $x^2 + 2x + 1$

$(x+1)(x+1)$

8. Descartes' Rule of Signs:

Suppose $P(x)$ is a polynomial whose terms are arranged in descending powers of a variable. Then the number of positive real zeros of $P(x)$ is the same as the number of changes in sign of the coefficients of the terms, or is less than this by an EVEN number.

The number of negative real zeros of $P(x)$ is the same as the number of changes in sign of the coefficients of the terms of $P(-x)$ or is less than this by an **EVEN** number.

9. State the number of complex zeros, the number of possible positive real zeros, the number of possible negative real zeros, and the number of possible imaginary zeros.

a. $P(x) = 2x^4 - x^3 - 2x^2 + 5x + 1$ 2 pos.

$P(-x) = 2x^4 + x^3 - 2x^2 - 5x + 1$ 2neg.

Pos	Neg
2 or 0	2 or 0

Have to subtract 2 until you can't to account for imaginary pairs

P	N	I
2	2	0
2	0	2
0	2	2
0	0	4

= 4 ↑ degree

b. $P(x) = 2x^3 + 3x^2 + 5x + 2$ 0 pos

$P(-x) = -2x^3 + 3x^2 - 5x + 2$

3 or 1 neg.

P	N	I
0	3	0
0	1	2

10. Rational Root Theorem:

If the polynomial $f(x) = a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0$ has integer coefficients, every rational zero of f has the form of $\frac{p}{q}$ (where $\frac{p}{q}$ is reduced), p is a factor of the constant term, a_0, and q is a factor of the leading coefficient, a_n.

You use the rational zero test to list all possible rational roots of a polynomial.

You use this list to find all rational roots of a polynomial, without a graphing calculator.

11. List all possible rational zeros of $f(x) = -x^4 + 3x^2 + 4$.

$p = 4$ $\pm 1, \pm 2, \pm 4$

$q = -1$ ± 1

$p/q = \pm 1, \pm 2, \pm 4$

12. List all possible rational zeros of $f(x) = 4x^5 + 12x^4 - x - 3$.

$p = -3$ $\pm 1, \pm 3$

$q = 4$ $\pm 1, \pm 2, \pm 4$

$p/q = \pm 1, \pm 1/2, \pm 3, \pm 3/2, \pm 1/4, \pm 3/4$

Homework: 4.3 and 4.4 Worksheet