4.3 and 4.4 Worksheet

I. Synthetic Division, Remainder Theorem, Factor Theorem

1. Determine k so that $f(x) = x^3 + kx^2 - kx + 10$ is divisible by $x + 3$.

   $-3 \,|\; 1 \;\; k \;\; -k \;\; 10$; $\;-3 \;\; -3k+9 \;\; 12k-27$; $\;1 \;\; k-3 \;\; -4k+9$ $\quad k = -17/12$

2. Determine all values of k so that $f(x) = k^2x^2 - 4kx + 3$ is divisible by $x - 1$.

   $1 \,|\; k^2 \;\; -4k \;\; +3$; $\;k^2$; $\;k^2 \;\; k^2-4k$ $\quad k^2-4k+3=0$ $\quad k=3 \;\; k=1$

3. Find k so that when $x^3 - kx^2 - kx + 1$ is divided by $x - 2$, the remainder = 0.

   $2 \,|\; 1 \;\; -k \;\; -k \;\; 1$; $\;2 \;\; 4-2k \;\; 8-6k$; $\;1 \;\; 2-k \;\; 4-3k$ $\quad 8-6k=-1$ $\quad -6k=-9$ $\quad k=3/2$

4. Find k so that when $x^3 - x^2 - kx + 10$ is divided $x - 3$, the remainder is -2.

   $3 \,|\; 1 \;\; -1 \;\; -k \;\; 10$; $\;3 \;\; 6 \;\; -3k+18=-12$; $\;1 \;\; 2 \;\; -k+6$ $\quad k=10$

5. Find the remainder if the polynomial $3x^{100} + 5x^{85} - 4x^{38} + 2x^{17} - 6$ is divided by $x + 1$.

   $3 \; -5 \; -4 \; -2 \; -6$ $\quad$ (14)

II. Use synthetic division to show that c is a zero of $f(x)$.

6. $f(x) = 3x^4 + 8x^3 - 2x^2 - 10x + 4$; c = -2

   $-2 \,|\; 3 \;\; 8 \;\; -2 \;\; -10 \;\; 4$; $\;-6 \;\; -4 \;\; 12 \;\; -4$; $\;3 \;\; 2 \;\; -6 \;\; 2 \;\; 0$

7. $f(x) = 4x^3 - 9x^2 - 8x - 3$; c = 3

   $3 \,|\; 4 \;\; -9 \;\; -8 \;\; -3$; $\;12 \;\; 9 \;\; 3$; $\;4 \;\; 3 \;\; 1 \;\; 0$

8. $f(x) = 4x^3 - 6x^2 + 8x - 3$; c = ½

   $1/2 \,|\; 4 \;\; -6 \;\; 8 \;\; -3$; $\;2 \;\; -2 \;\; 3$; $\;4 \;\; -4 \;\; 6 \;\; 0$

9. Show that –3 is a zero of multiplicity 2 of the polynomial function $P(x) = x^4 + 7x^3 + 13x^2 - 3x - 18$ and express P(x) as a product of linear factors.

   $-3 \,|\; 1 \;\; 7 \;\; 13 \;\; -3 \;\; -18$; $\;1 \;\; 4 \;\; 1 \;\; -6 \;\; 0$ $\quad -3 \,|\; 1 \;\; 4 \;\; 1 \;\; -6$; $\;1 \;\; 1 \;\; -2 \;\; 0$ $\quad x^2+x-2$

   $P(x) = (x+3)^2(x+2)(x-1)$

10. Show that –1 is a zero of $f(x)$ with multiplicity of 4 if $f(x) = x^5 + x^4 - 6x^3 - 14x^2 - 11x - 3$. Express $f(x)$ as a product of linear factors.

    $f(x) = (x+1)^4(x-3)$

11. Find a polynomial function of degree 4 such that both –2 and 3 are zeros of multiplicity 2.

    $f(x) = (x+2)^2(x-3)^2 = x^4 - 2x^3 - 11x^2 + 12x + 36$

12. Find a polynomial function of degree 5 such that –2 is a zero of multiplicity 3 and 4 is a zero of multiplicity 2.

    $P(x) = x^5 - 2x^4 - 20x^3 + 8x^2 + 128x + 128$

III. List all possible rational roots for each equation.

13. $f(x) = 2x^4 + 7x^3 + 5x - 4$

    $p = -4 \;\; \pm1, \pm2, \pm4$; $q = 2 \;\; \pm1, \pm2$ $\quad \pm1, \pm1/2, \pm2, \pm4$

14. $g(x) = 3x^3 - 8x^2 + 9x - 6$

    $p = -6 \;\; \pm1, \pm2, \pm3, \pm6$; $q = 3 \;\; \pm1, \pm3$ $\quad \pm1, \pm1/3, \pm2, \pm2/3, \pm3, \pm6$

IV. Descartes Rule of Signs:

Complete the chart:

	Total Complex Roots	Number of possible +	Number of possible -	Number of possible imaginary
14. $y = -x^3 + x^2 - x + 1$	3	3 or 1	0	0 or 2
15. $y = 3x^4 + 2x^3 - 3x^2 - 4x + 1$	4	2 or 0	2 or 0	0 or 2 or 4.
16. $y = -7x^3 - 6x + 1$	3	1	0	2
17. $y = x^{10} - x^8 + x^6 - x^4 + x^2 - 1$	10	5 or 3 or 1	5 or 3 or 1	0 or 2 or 4 or 6 or 8
18. $y = x^5 - x^3 - x + 1$	5	2 or 0	1	2 or 4
19. $y = 3x^4 - x^2 + x - 1$	4	3 or 1	1	0 or 2

V. Find all rational roots using the rational root theorem and Descartes Rule to help.

20. $f(x) = x^3 - 7x^2 + 7x + 15$

    $-1 \,|\; 1 \;\; -7 \;\; 7 \;\; 15$; $\;-1 \;\; 8 \;\; -15$; $\;1 \;\; -8 \;\; 15 \;\; 0$

    $x = -1$

    $x = 5$

    $x = 3$