11.6 Exponential Equations

Solve over the set of real numbers. Express each solution to three significant digits.

1. $\left(\frac{1}{3}\right)^{x-6} = 3^x$ = 3

$(3^{-1})^{x-6} = 3^x$

$3^{-x+6} = 3^x$

$-x+6 = x$

$6 = 2x$

$3 = x$

2. $\left(\frac{1}{8}\right)^{x} = 2^{x-6}$ = 3/2

$2^{-3x} = 2^{x-6}$

$-3x = x-6$

$-4x = -6$

$x = 6/4 = 3/2$

3. $9^{3x} = 27^{x+2}$ = 2

$3^{6x} = 3^{3x+6}$

$6x = 3x+6$

$3x = 6$

$x = 2$

4. $9^x = 45$ = 1.732

$\log 9^x = \log 45$

$x \cdot \log 9 = \log 45$

$x = \frac{\log 45}{\log 9}$

or $\log_9 45$

5. $5^x = 52$ = 2.455

$\log 5^x = \log 52$

$x \log 5 = \log 52$

$x = \frac{\log 52}{\log 5}$

or $\log_5 52$

6. $4^{3p} = 10$ = .5537

$\log 4^{3p} = \log 10$

$3p \log 4 = \log 10$

$p = \frac{\log 10}{3 \log 4}$

or $\frac{1}{\log_4 10}$

7. $3^{n+2} = 14.5$ = .4341

$(n+2)\log 3 = \log 14.5$

$n \log 3 + 2\log 3 = \log 14.5$

$n \log 3 = \log 14.5 - 2\log 3$

$n = \frac{\log 14.5 - 2\log 3}{\log 3}$

8. $9^{z-4} = 6.28$ = 4.836

$(z-4)\log 9 = \log 6.28$

$z \log 9 - 4\log 9 = \log 6.28$

$z \log 9 = \log 6.28 + 4\log 9$

$z = \frac{\log 6.28 + 4\log 9}{\log 9}$

9. $8.2^{n-3} = 42.5$ = 4.782

$(n-3)\log 8.2 = \log 42.5$

$n \log 8.2 - 3\log 8.2 = \log 42.5$

$n \log 8.2 = \log 42.5 + 3\log 8.2$

$n = \frac{\log 42.5 + 3\log 8.2}{\log 8.2}$

10. $2.1^{t-5} = 9.32$ = 8.009

$(t-5)\log 2.1 = \log 9.32$

$t\log 2.1 - 5\log 2.1 = \log 9.32$

$t\log 2.1 = \log 9.32 + 5\log 2.1$

$t = \frac{\log 9.32 + 5\log 2.1}{\log 2.1}$

11. $5^{x^2} = 68$ = ±1.619

$x^2 \log 5 = \log 68$

$x^2 = \frac{\log 68}{\log 5}$

$x = \pm\sqrt{\frac{\log 68}{\log 5}}$

12. $5^x = 3^x$ = 0

$x \log 5 = x \log 3$

$x \log 5 - x \log 3 = 0$

$x = 0$

13. $3^{x+1} = 7^{x-2}$ = 5.89

$(x+1)\log 3 = (x-2)\log 7$

$x\log 3 + \log 3 = x\log 7 - 2\log 7$

$x\log 3 - x\log 7 = -2\log 7 - \log 3$

$x(\log 3 - \log 7) = -2\log 7 - \log 3$

$x = \dfrac{-2\log 7 - \log 3}{\log 3 - \log 7}$

14. $2^{x-1} = 6^{2x+3}$ = -2.1

$(x-1)\log 2 = 2x+3(\log 6)$

$x\log 2 - \log 2 = 2x\log 6 + 3\log 6$

$x\log 2 - 2x\log 6 = 3\log 6 + \log 2$

$x(\log 2 - 2\log 6) = 3\log 6 + \log 2$

$x = \dfrac{3\log 6 + \log 2}{\log 2 - 2\log 6}$

15. $x = \log_4 19.5$ = 2.143

$x = \dfrac{\log 19.5}{\log 4}$

16. $\log_{27} \dfrac{1}{3} = x$ = -1/3

$27^x = \frac{1}{3}$

$3^{3x} = 3^{-1}$

$3x = -1$

$x = -1/3$

17. $5a^{\frac{2}{5}} = 15.35$ = 16.514

$a^{2/5} = 3.07$

$a = 3.07^{5/2}$

18. $3x^{\frac{4}{3}} = 21.3$ = 4.350

$x^{4/3} = 7.1$

$x = 7.1^{3/4}$

Homework: page 639 #16, 17, 19-21, 23, 25-30, 32, 35, 36