Ex: Use the power rule to find the derivative of $f(x) = \frac{1}{x}$?

Ex: How do we find the derivative of $f(x) = \sin(x)$?

Ex: Write the equation of the tangent line of $f(x) = \sqrt[3]{x}$ at x = 2.

Ex: Use the limit definition to find the derivative of $f(x) = \cos(x)$?

Ex: Use the limit definition to find the derivative of $f(x) = \tan(x)$?

$$\frac{\tan(x+h) - \tan x}{h} = \frac{\tan x + \tan h}{1 - \tan x \tan h} - \tan x \;\Bigg|\; \boxed{\frac{1 - \tan x \tan h}{1 - \tan x \tan h}}$$

$$\frac{1}{h} \cdot \frac{\cancel{\tan x} + \tan h - \cancel{\tan x} + \tan^2 x \tan h}{1 - \tan x \tan h} = \frac{\tan h}{h} \frac{(\sec^2 x)}{1 - \tan x \tan h} = \frac{\sec^2 x}{1}$$

Ex: Use the limit definition to find the derivative of $f(x) = \sec(x)$?

$$\frac{\sec(x+h) - \sec(x)}{h} \quad \frac{1}{\cos(x+h)} - \frac{1}{\cos(x)} = \frac{\cos(x) - \cos(x+h)}{\cos(x) \cdot \cos(x+h)} \cdot \frac{1}{h}$$

$$\frac{\cos x + [-\cos x \cos h + \sin x \sin h]}{h \cos(x)(\cos x \cos h - \sin x \sin h)} = \frac{\sin x \sin h}{\cos x\, h\,(\cos x \cos h - \sin x \sin h)} + \cos x \frac{(1 - \cos h)}{'' \; '' \; h}$$

$$= \frac{\sin x}{\cos^2 x} = \tan x \cdot \sec x$$

Ex: Use the limit definition to find the derivative of $f(x) = \csc(x)$?

Day 3 Notes:

Practice the Power Rule: Find the derivative of each of the following.

Ex 1: $f(x) = 4x^{12}$

$48x^{11}$

Ex 2: $f(x) = 3x^{\frac{7}{4}}$

$21x^{3/4}$

Ex 3: $f(x) = \frac{5}{x^3}$ $\quad 5x^{-3}$

$-15x^{-4} = \frac{-15}{x^4}$

Ex 4: $f(x) = -3\sqrt[4]{x} + 2x^{-3}$

$-3x^{-3/4} - 6x^{-4}$

$\frac{-3}{\sqrt[4]{x^3}} - \frac{6}{x^4}$

Ex 5: $f(x) = \frac{2}{3}x^{\frac{3}{2}} - \frac{3}{4}x^{\frac{3}{5}}$

$x^{1/2} - \frac{9}{20}x^{-2/5}$

$\sqrt{x} - \frac{9}{20\sqrt[5]{x^2}}$

Ex 6: $f(x) = \sqrt{2}\,\sqrt[3]{x} + \sqrt{2}\,\sqrt[5]{x}$

$\frac{\sqrt{2}}{3}x^{-2/3} + \frac{\sqrt{2}}{5}x^{-4/5}$

$\frac{\sqrt{2}}{3\sqrt[3]{x^2}} + \frac{\sqrt{2}}{5\sqrt[5]{x^4}}$

The Product Rule: $\frac{d}{dx}[f(x) \bullet g(x)] = f(x)g'(x) + g(x)f'(x)$

Ex 7: $f(x) = (2x - 3x^3)(5x - 4)$

$(2x - 3x^3)\cdot 5 + (5x-4)(2-9x^2)$

$10x - 15x^3 + 10x - 45x^3 - 8 + 36x^2$

$-60x^3 + 36x^2 + 20x - 8$

Ex 8: $f(x) = 3\sin(x)\cos(x)$

$3\sin(x)\cdot(-\sin x) + 3\cos x(\cos x)$

$-3\sin^2 x + 3\cos^2 x$

Ex 9: $f(x) = \frac{2}{x^4}\tan(x)$

$2x^{-4}\cdot(+\sec^2 x) - \tan x(-8x^{-5})$

$\frac{2\sec^2 x}{x^4} + \frac{8\tan x}{x^5}$

$\frac{2x\sec^2 x + 8\tan x}{x^5}$

What would be the second derivative ($f''(x)$ or $\frac{d^2y}{dx^2}$) of example # 7?

$y' = -60x^3 + 36x^2 + 20x - 8$

$y'' = -180x^2 + 72x + 20$

The Quotient Rule: $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$ **(Low D High – High D Low)**

Ex 10: $f(x) = \frac{x^2 + 3x}{2x + 1}$

$\frac{(2x+1)(2x+3) - (x^2+3x)\cdot 2}{(2x+1)^2}$

$\frac{4x^2 + 8x + 3 - 2x^2 - 6x}{(2x+1)^2}$

$\frac{2x^2 + 2x + 3}{(2x+1)^2}$

Ex 11: $f(x) = \cot(x)$

$\frac{\cos x}{\sin x}$

$\frac{\sin x(-\sin x) - \cos x(\cos x)}{\sin^2 x}$

$\frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-1}{\sin^2 x}$

$= -\csc^2 x$

Ex 12: $f(x) = \frac{\sqrt{x} + 3}{\sqrt{x} - 3}$

$\frac{(\sqrt{x}-3)\frac{1}{2}x^{-1/2} - [(\sqrt{x}+3)\frac{1}{2}x^{-1/2}]}{(\sqrt{x}-3)^2}$

$\frac{\frac{\sqrt{x}-3}{2\sqrt{x}} - \frac{\sqrt{x}+3}{2\sqrt{x}}}{(\sqrt{x}-3)^2} = \frac{\frac{-3}{\sqrt{x}}}{(\sqrt{x}-3)^2}$

Practice: Find the derivatives of each.

Ex 13: $f(x) = \sqrt[3]{x^2}\sec(x)$

$x^{2/3}\sec x\tan x + \frac{2}{3}x^{-1/3}\sec x$

$x^{2/3}\sec x\tan x + \frac{2}{3\sqrt[3]{x}}\sec x$

Ex 14: $f(x) = \frac{\sqrt{x+2}}{x^3}$

$\frac{x^3\left(\frac{1}{2}(x+2)^{-1/2}\right) - \sqrt{x+2}\cdot 3x^2}{x^6}$

$\frac{\frac{x^3}{2\sqrt{x+2}} - 3x^2\sqrt{x+2}\ \boxed{\frac{2\sqrt{x+2}}{2\sqrt{x+2}}}}{x^6}$

$\frac{-5x^3 - 12x^2}{2x^6\sqrt{x+2}} =$

Ex 15: $f(x) = \frac{3 - \frac{1}{x}}{x+5} = \frac{\frac{3x-1}{x}}{x+5}$

$\frac{3x-1}{x^2+5x} = \frac{(x^2+5x)3 - (3x-1)(2x+5)}{(x^2+5x)^2}$

$\frac{3x^2+15x-6x^2-13x+5}{(x^2+5x)^2} = \frac{-3x^2+2x+5}{(x^2+5x)^2}$

Ex 16: $f(x) = \frac{1-\cos(x)}{\sin(x)}$ (Use both forms)

$f'(x) = \frac{\sin x(\sin x) - (1-\cos x)(\cos x)}{\sin^2 x}$

$= \frac{\sin^2 x - \cancel{\cos x} + \cos^2 x}{\sin^2 x}$

$= \frac{1-\cos x}{\sin^2 x}$

$f(x) = \csc x - \cot x$

$f'(x) = -\csc x\cot x + \csc^2 x$

$= \frac{-1}{\sin x}\cdot\frac{\cos x}{\sin x} + \frac{1}{\sin^2 x}$

$= \frac{1-\cos x}{\sin^2 x}$

* What would you do here? Find the derivative of $f(x) = \frac{x\sqrt{1+x}}{x^3\sin(x)}$

$f(x) = x\sqrt{1+x}$ $\quad f'(x) = \frac{x}{2\sqrt{1+x}} + \sqrt{1+x}$

$g(x) = x^3\sin(x)$ $\quad g'(x) = x^3\cos x + 3x^2\sin x$

$\frac{x^3\sin x\left(\frac{x}{2\sqrt{1+x}} + \sqrt{1+x}\right) - x\sqrt{1+x}\left(x^3\cos x + 3x^2\sin x\right)}{x^6\sin^2(x)}$

Higher Order Derivatives:

Ex ~~16~~ 17: $f(x) = -x^2$ Find $\frac{d^2y}{dx^2}$

$\frac{dy}{dx} = -2x$

$\frac{d^2y}{dx^2} = -2$

Ex ~~17~~ 18: $f(x) = -2x^3 - 4x^{-3}$ Find $\frac{d^3y}{dx^3}$

$y' = -6x^2 + 12x^{-4}$

$y'' = -12x - 48x^{-5}$

$y''' = -12 + 240x^{-6}$

Ex ~~18~~ 19: $f(x) = -x^4 + 2\sqrt[5]{x^3}$ Find y'''

$x^{3/5}$

$y' = -4x^3 + \frac{6}{5}x^{-2/5}$

$y'' = -12x^2 - \frac{12}{25}x^{-7/5}$

$y''' = -24x + \frac{84}{125}x^{-12/5}$

Ex ~~19~~ 20: $f(x) = \cos(x) - 3x^{-2}$ Find y'''

$y' = -\sin(x) + 6x^{-3}$

$y'' = -\cos(x) - 18x^{-4}$

$y''' = \sin(x) + 72x^{-5}$

Ex ~~18~~ 21: $f(x) = -x^2\cos(x) + \frac{1}{2x^3}$ Find $\frac{d^3y}{dx^3}$

$f'(x) = -x^2\cdot -\sin x + -2x(\cos x) - \frac{3}{2}x^{-4}$

$x^2\sin x - 2x\cos x - \frac{3}{2}x^{-4}$

$f''(x) = x^2(-\cos x) + 2x\sin x - 2x(-\sin x) + (\cos x)\cdot 2 + 6x^{-5}$

$-x^2\cos x + 4x\sin x - 2(\cos x) + 6x^{-5}$

$f'''(x) = x^2\sin x - 2x\cos x + 4x(\cos x) + 4(-\sin x) + 2\sin x - 30x^{-4}$

$= x^2\sin x + 2x\cos x - 2\sin x - 30x^{-4}$