Inverse Functions:

Let f and g be two functions such that $f(g(x)) = x$ for every x in the domain of g and $g(f(x)) = x$ for every x in the domain of f.

The function g is the inverse of the function f, and is denoted by f^{-1} (read "f-inverse").

Thus, $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$

The domain of f is equal to the range of f^{-1}, and vice versa.

The graph of an inverse is the reflection of the original function over the line y = x.

I. Verify that the following are inverses: Show that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

1. $f(x) = 2x + 3$

$g(x) = \dfrac{x-3}{2}$

$f(g(x)) = 2\left(\frac{x-3}{2}\right) + 3 = x$

$g(f(x)) = \frac{2x+3-3}{2} = x$

2. $f(x) = \sqrt{3x+5}$

D: $[-5/3, \infty)$

R: $[0, \infty)$

$g(x) = \dfrac{x^2 - 5}{3}$ $x \geq 0$

1. To have an ***inverse function***, a function must be one-to-one, which means no two elements in the domain correspond to the same element in the range of f.

You can use the horizontal line test to determine if a function is one-to-one.

2. If f is strictly monotonic on its entire domain, then it is one-to-one and therefore has an inverse function.

A function is monotonic if it is either strictly increasing or decreasing on its entire domain.

Do the following functions have an inverse function? Use the derivative .

Ex: $f(x) = \sqrt{x+2}$ $(-2, \infty)$

$f'(x) = \frac{1}{2}(x+2)^{-1/2}$

$\frac{1}{2\sqrt{x+2}} > 0$ increasing everywhere on domain ∴ monotonic

It has an inverse function

Ex: $g(x) = -\frac{1}{6}x^3 - 5x + 4$ $[0, \infty)$

$g'(x) = -\frac{1}{2}x^2 - 5$

decreasing everywhere

g^{-1} is a function

Ex: $h(x) = \sin x$ $\left(\pi, \frac{3\pi}{2}\right)$

$h'(x) = \cos x$

$\cos x < 0$

decreasing

h^{-1} is a function.

Ex: $p(x) = \frac{1}{x}$

$p'(x) = \frac{-1}{x^2}$

$\frac{-1}{x^2} < 0$

p^{-1} is a function.

$x \geq 0$

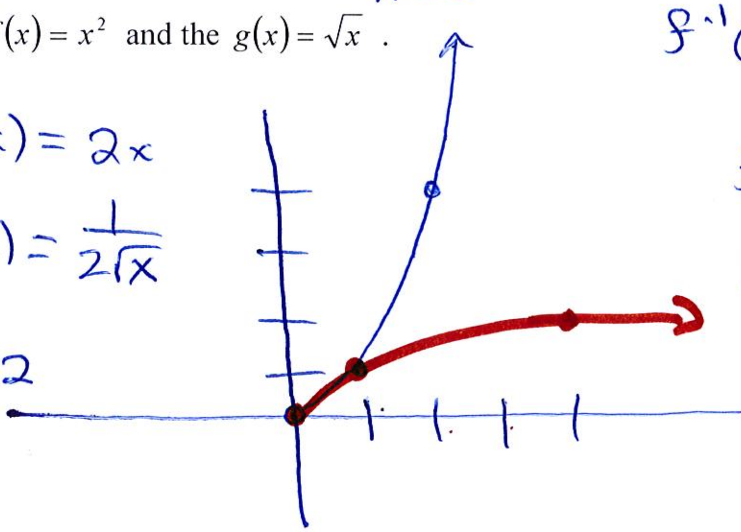
Discuss $f(x) = x^2$ and the $g(x) = \sqrt{x}$.

$f^{-1}(x) = g(x)$

$f'(x) = 2x$

$g'(x) = \frac{1}{2\sqrt{x}}$

$x = 2$



$f'(2) = 4$

$g'(4) = 2$

Ex: $f(x) = \sqrt{x^3 + 1}$ Find $(f^{-1})'(3) =$

$f'(x) = \frac{1}{2}(x^3+1)^{-1/2} \cdot 3x^2$

what is x in f(x) that gives us 3 ?

$3 = \sqrt{x^3+1}$

$9 = x^3 + 1$

$8 = x^3$

$2 = x$

$(f^{-1})'(3) = \frac{1}{f'(f^{-1}(3))}$

$= \frac{1}{f'(2)}$

$= \frac{1}{2}$

Ex: $f(x) = \ln x$ Find $(f^{-1})'(1) =$

$f'(x) = \frac{1}{x}$

What is x in f(x) that gives us 1

$\ln x = 1 \quad x = e$

$(f^{-1})'(1) = \frac{1}{f'(f^{-1}(1))}$

$= \frac{1}{f'(e)}$

$= \frac{1}{\frac{1}{e}}$

$= e$