Day 2 Sequences and Series Notes

1. **Recursive Formula:**
A formula for a sequence that gives the value of a term t_n in terms of the preceding term t_{n-1}. The first term is represented by t_1, the second term in represented by t_2, the third term in represented by t_3, and so forth.

Explicit Formula: $\boxed{a_n = a_{n-1} + d}$ or $\boxed{a_{n+1} = a_n + d}$ **if Arithmetic Sequence!**

2. Find the next three terms in each sequence.

a. 80, 77, 74, 71, 68, … b. 4, 8, 16, 32, 64, … *c. 0, 3, 7, 12, 18, … * d. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \ldots$

65, 62, 59 128, 256, 512 $\frac{1}{64}, \frac{1}{128}, \frac{1}{256}$

$a_{n+1} = a_n - 3$ $a_{n+1} = 2a_n$ $a_{n+1} = a_n + (\quad)$ $a_{n+1} = \frac{1}{2}a_n$

3. Now write the recursive formula for the sequences above.

$a_{n+1} = a_n + (n+2)$

4. If $a_1 = 22$ and $a_n = a_{n-1} - 3$, find the next three terms.

$a_2 = a_{2-1} - 3$ $a_3 = a_{3-1} - 3$ $a_4 = a_{4-1} - 3$

$a_2 = 22 - 3$ $a_3 = 19 - 3$ $a_4 = a_3 - 3$

$a_{22} = 19$ $a_3 = 16$ $a_4 = 13$

5. If $t_1 = 64$ and $t_n = \frac{1}{2}t_{n-1}$, find the next four terms.

$t_2 = \frac{1}{2}t_{2-1}$

$t_2 = \frac{1}{2}t_1$

$t_2 = 32$ $t_3 = 16$ $t_4 = 8$ $t_5 = 4$

6. If $a_1 = 3$, $a_2 = 5$ and $a_n = a_{n-2} + 4a_{n-1}$, find the third, fourth and fifth terms..

$a_3 = a_{3-2} + 4a_{3-1}$

$a_3 = a_1 + 4a_2 = 3 + 4(5) = 23$

$a_4 = a_2 + 4a_3 = 5 + 4(23) = 97$

$a_5 = a_3 + 4a_4 = 23 + 4(97) = 411$

7. Find the first four terms.

a. $t_n = 16 - 3n$ b. $a_k = 3 + 4(k-1)$ c. $a_n = \frac{1}{n} + \frac{1}{n+2}$

$t_1 = 16 - 3(1) = 13$
$t_2 = 16 - 3(2) = 10$
$t_3 = 16 - 3(3) = 7$
$t_4 = 16 - 3(4) = 4$

$a_1 = 3 + 4(1-1) = 3$
$a_2 = 3 + 4(2-1) = 7$
$a_3 = 3 + 4(3-1) = 11$
$a_4 = 3 + 4(4-1) = 15$

$a_1 = \frac{1}{1} + \frac{1}{1+2} = \frac{4}{3}$
$a_2 = \frac{1}{2} + \frac{1}{2+2} = \frac{3}{4}$
$a_3 = \frac{1}{3} + \frac{1}{3+2} = \frac{8}{15}$
$a_4 = \frac{1}{4} + \frac{1}{4+2} = \frac{5}{12}$

8. If the domain values are $\{1, 0, 3, 5\}$, find the corresponding range values for $t_n = 2n - 5$.

$t_1 = 2(1) - 5 = -3 \quad (1, -3)$

$t_0 = 2(0) - 5 = -5 \quad (0, -5)$

$t_3 = 2(3) - 5 = 1 \quad (3, 1)$

$t_5 = 2(5) - 5 = 5 \quad (5, 5)$

10. Sigma Notation
Simplifies the process of writing out the sum of a series

$$\sum_{n=2}^{10}(2n)$$

last value of n

first value of n

is read as the sum of $2n$ **as** n **increases from** 1 **to** 10

11. $\sum_{k=1}^{5}(2k)$

$2(1) + 2(2) + 2(3) + 2(4) + 2(5)$

$= 30$

12. $\sum_{k=-1}^{2}(k+2)$

$2 - (-1) = 3 + 1$

$S_n = \frac{4}{2}(1 + 4)$

$(-1+2) + (0+2) + (1+2) + (2+2)$

$= 10$

13. $\sum_{k=0}^{3} k + 2$

(no parentheses, careful)

$(0 + 1 + 2 + 3)$ then $+ 2$

$= 8$

14. $\sum_{k=2}^{5} 2^k$ ← can't do S_n (Geometric)

$2^2 + 2^3 + 2^4 + 2^5$

$= 60$

15. $\sum_{n=1}^{3} n^2 - \sum_{n=2}^{5} n$

$(1^2 + 2^2 + 3^2) - (2 + 3 + 4 + 5)$

$14 - 14 = 0$

16. $\sum_{k=5}^{9}[3 - 4(n-1)]$

$3 - 4(4) + 3 - 4(5) + 3 - 4(6)$
$+ 3 - 4(7) + 3 - 4(8)$

$= -105$

Homework: Worksheet #1 and Worksheet #2