

Day 2 Notes:

When do we know something definite?

$$\left\{ \begin{array}{l} \underline{\hspace{2cm}} + \underline{\hspace{2cm}} + \underline{\hspace{2cm}} = 5 \\ \underline{\hspace{2cm}} + \underline{\hspace{2cm}} + \underline{\hspace{2cm}} = 0 \\ \underline{\hspace{2cm}} \bullet \underline{\hspace{2cm}} \bullet \underline{\hspace{2cm}} = 5 \\ \underline{\hspace{2cm}} \bullet \underline{\hspace{2cm}} \bullet \underline{\hspace{2cm}} = 0 \end{array} \right. \text{ (something must equal zero)}$$

Zero Product Property :

Solve each of the following: All trig answers are $0 < \theta \leq 2\pi$ and may need a calculator.

Ex: $4x^2 + 4x - 15 = 0$

$(2x+5)(2x-3) = 0$
 $x = -5/2 \quad x = 3/2$

Ex: $2x^5 + 24x = 14x^3$

$2x(x^4 - 7x^2 + 12)$
 $2x(x^2 - 4)(x^2 + 3)$
 $x = 0 \quad x = \pm 2 \quad x = \pm i\sqrt{3}$

Ex: $x^2 + 7 = 0$

$x^2 = -7$
 $x = \pm i\sqrt{7}$

Ex: $4x^4 + 20x^2 = -25$

$(2x^2+5)(2x^2+5)$
 $x = \pm \frac{i\sqrt{10}}{2}$ (mult of 2)

Ex: $x^4 + x^3 - x = 1$

$x^3(x+1) - 1(x+1)$
 $(x+1)(x-1)(x^2+x+1)$
 $x = -1 \quad x = 1 \quad x = \frac{-1 \pm i\sqrt{3}}{2}$

Ex: $2x^6 - 16 = 0$

$2(x^6 - 8)$
 $2(x^2 - 2)(x^4 + 2x^2 + 4)$
 $x = \pm i\sqrt{2}$

Ex: $2\cos^2(\theta) - \cos(\theta) = 1$

$2\cos^2\theta - \cos\theta - 1 = 0$
 $(2\cos\theta + 1)(\cos\theta - 1) = 0$
 $\theta = 2\pi/3, 4\pi/3 \quad \theta = 2\pi$

Ex: $\csc^2(\theta) - 4 = 0$

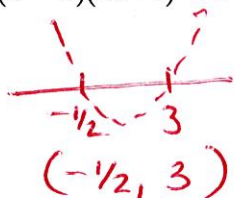
$(\csc\theta + 2)(\csc\theta - 2) = 0$
 $\csc\theta = -2 \quad \csc\theta = 2$
 $\theta = 7\pi/6, 11\pi/6 \quad \theta = \pi/6, 5\pi/6$

Ex: $36\sin^2(\theta) + 33\sin(\theta) = 20$

$(12\sin\theta - 5)(3\sin\theta + 4) = 0$
 $\sin\theta = 5/12 \quad \sin\theta = -4/3$
 $\theta = \arcsin(5/12)$ CAN'T

Inequalities: All answers must be in Interval Notation.

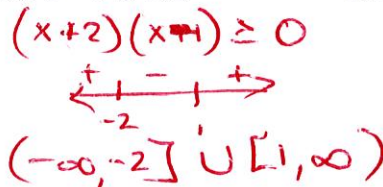
Ex: $(x - 3)(2x + 1) < 0$



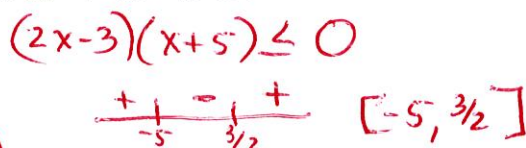
Ex: $(x - 5)(x - 2)(x + 1) \geq 0$

$[-1, 2] \cup [5, \infty)$

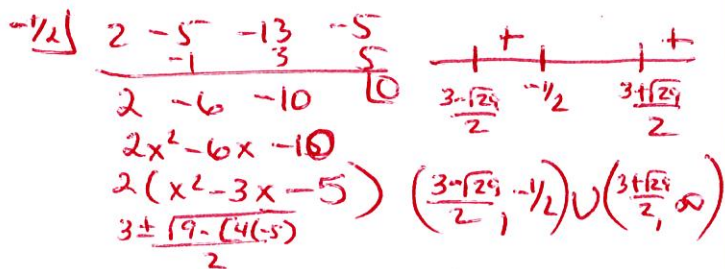
Ex: $x^2 + x \geq 2$



Ex: $2x^2 + 7x - 15 \leq 0$



Ex: $2x^3 - 5x^2 - 13x > 5$



Absolute Value Inequalities: \geq "great" or "less than" \leq "less than"

Ex: $|x| < 3$

$x < 3$ and $x > -3$
 $(-3, 3)$

Ex: $|x| \geq 2$

$x \geq 2$ or $x \leq -2$
 $(-\infty, -2] \cup [2, \infty)$

Ex: $|x-2| < 3$

$x-2 < 3$ and $x-2 > -3$
 $x < 5$ AND $x > -1$
 $(-1, 5)$

Ex: $|x-2| < 5$

$x-2 < 5$ and $x-2 > -5$
 $x < 7$ AND $x > -3$
 $(-3, 7)$

Ex: $|2x+7| < 11$

$2x+7 < 11$ and $2x+7 > -11$
 $2x < 4$ AND $2x > -22$
 $(-11, 2)$

Ex: $|3x-2| > 8$

$3x-2 > 8$ and $3x-2 < -8$
 $x > 10/3$ or $x < -2$
 $(-\infty, -2) \cup (10/3, \infty)$

Ex: $|\frac{1}{5}x-2| < 4x$

$\frac{1}{5}x-2 < 4x$ AND $\frac{1}{5}x-2 > -4x$
 $-2 < 19/5x$ and $2/5x > 2$
 $-10/19 < x$ AND $x > 10/21$
 $(10/21, \infty)$

Ex: $|3-4x| > 9$

$3-4x > 9$ or $3-4x < -9$
 $-4x > 6$ or $-4x < -12$
 $x < -3/2$ or $x > 3$
 $(-\infty, -3/2) \cup (3, \infty)$

Ex: $|4x|-18 < -1$

$4x < 17$ and $4x > -17$
 $x < 17/4$ AND $x > -17/4$
 $(-17/4, 17/4)$

Ex: $|3x+1| \geq 2x+5$

$3x+1 \geq 2x+5$ or $3x+1 \leq -2x-5$
 $x \geq 4$ or $x \leq -6/5$
 $(-\infty, -6/5] \cup [4, \infty)$

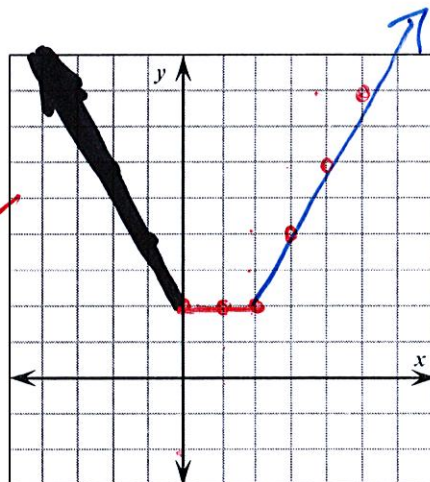
Ex: $|a-b|$ What does this represent?

distance from a to b

Write $f(x) = |x| + |x-2|$ as a piecewise and graph.

$$f(x) = \begin{cases} -2x+2 & \text{if } x \leq 0 \\ 2 & \text{if } 0 < x < 2 \\ 2x-2 & \text{if } x \geq 2 \end{cases}$$

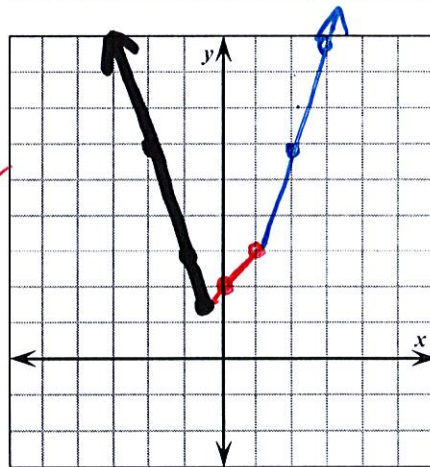
Change Both
 $-x + (-x+2)$ $x + (-x+2)$ $x + x - 2$
 $|x|$ change same same
 $|x-2|$ change change same, critical pts.



Write $f(x) = |2x+1| + |x-1|$ as a piecewise and graph.

$$f(x) = \begin{cases} -3x & \text{if } x < -1/2 \\ x+2 & \text{if } -1/2 \leq x \leq 1 \\ 3x & \text{if } x > 1 \end{cases}$$

$-2x-1 + -x+1$ $2x+1 + -x+1$ $2x+1 + x-1$
 $|2x+1|$ change same same
 $|x-1|$ change change same



Remember $\sqrt{6x+12} = 1 + \sqrt{4x+9}$ creates Extraneous Answers!

$6x+12 = 1 + 2\sqrt{4x+9} + 4x+9$

$2x+2 = 2\sqrt{4x+9}$

$x+1 = \sqrt{4x+9}$

$x^2+2x+1 = 4x+9$

$x^2-2x-8 = 0$

$(x-4)(x+2) = 0$
 $x=4$ $x=-2$

$x \neq -2$
 Extra.