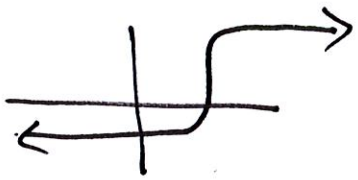
If f is differentiable at x = c, then f is continuous at x = c.

Vertical Tangents:



Day 2 Notes: Power Rule:

Ex: What is the derivative of $f(x) = -2$? 0

Ex: What is the derivative of $f(x) = 3x$? 3

Ex: Use the limit definition to find the derivative of $f(x) = x^2$?

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$\frac{(x+h)^2 - x^2}{h} = \frac{x^2 + 2xh + h^2 - x^2}{h} = 2x$$

Ex: Use the limit definition to find the derivative of $f(x) = x^3$?

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{(x+h)^3 - x^3}{h} = \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$$

$$= 3x^2$$

Ex: Use the limit definition to find the derivative of $f(x) = x^4$?

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - x^4}{h}$$

$$= 4x^3$$

Ex: Use the limit definition to find the derivative of $f(x) = x^5 + 3x^2 - 6x$?

Power x coefficient and exponent goes down by 1.

$$f'(x) = 5x^4 + 6x^1 - 6$$

Ex: Use the limit definition to find the derivative of $f(x) = \sqrt{x}$?

$$x^{1/2}$$

$$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

1 5 10 10 5 1

1 6 15 20 15 6 1

Ex: Use the limit definition to find the derivative of $f(x)=\frac{1}{x}$?

x^{-1}

$-x^{-2} = \frac{-1}{x^2}$ $\quad x=3 \quad \frac{-1}{9}$

Ex: Use the limit definition to find the derivative of $f(x)=\sin(x)$?

$$\lim_{h\to 0}\frac{\sin(x+h)-\sin(x)}{h} = \cos(x)$$

$$\frac{\sin x\cos h+\cos x\sin h-\sin(x)}{h}$$

$$\frac{\sin x(\cos h-1)}{h} \qquad \frac{\cos x \sin h}{h}$$

Ex: Write the equation of the tangent line of $f(x)=\sqrt[3]{x}$ at x = 2.

$x^{1/3}$

$\frac{1}{3}x^{-2/3} \qquad x=2$

$\frac{1}{3\sqrt[3]{x^2}} = \frac{1}{3\sqrt[3]{4}}$ $\quad (2, \sqrt[3]{2})$

Ex: Use the limit definition to find the derivative of $f(x)=\cos(x)$?

$$\frac{\cos x\cos h-\sin x\sin h-\cos(x+h)-\cos x}{h}$$

$$\frac{\cos x\cos h-\sin x\sin h-\cos x}{h}$$

$f'(x) = -\sin x$