GCF
Binomials
Trinomials $a=1$ $1x^2-2x-24$ simple
Trinomials $16x^2+2x-5$ prod -80
Group sum $10,-8$

Day 2

4. Factor $f(x) = x^3 - 2x^2 - 4x + 8$ and sketch its graph.

$(2x-1)(8x+5)$

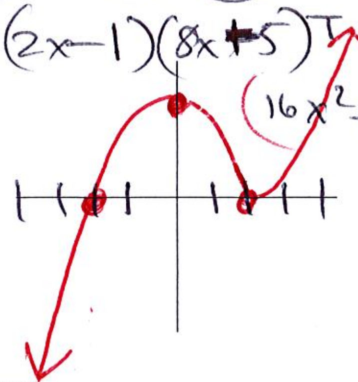
$(16x^2+10x)-8x-5$

$(x^3-2x^2)[-4x+8]$

$x^2(x-2)-4(x-2)$

$(x^2-4)(x-2)$

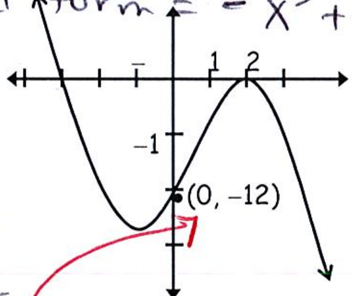
$(x+2)(x-2)(x-2)$

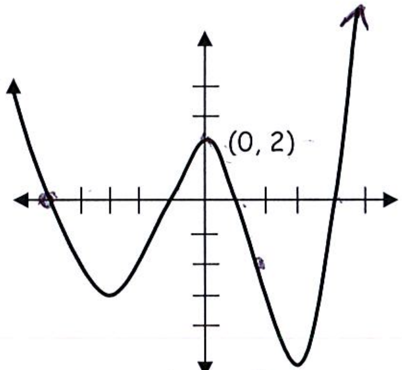


5. Write the equation for the polynomial graph shown below.

factored form $= -(x+3)(x-2)^2$

general form $= -x^3+x^2+8x-12$





x^2-4x+4
$x+3$
$3x^2-12x+12$
x^3-4x^2+4x
$x^3-x^2-8x+12$

$y=a(x+3)(x-2)^2$

$12=a(3)(4)$ $a=-1$

$y=a(x+4)(x+1)(x-1)(x-4)$

$2=a(4)(1)(-1)(-4)$

$a=1/8$

6. Find a polynomial function that has the given zeros.

a. 0,2,7 b. 4,-3,3 c. -2, -1, 3, 2

$x(x-2)(x-7)$ $(x-4)(x+3)(x-3)$

Remainder Theorem — for $f(x)$, the value of $f(a)$ is equal to the remainder when $f(x)$ is divided by $x - a$.

Ex: $f(x) = 3x^4 + x^3 - 2x^2 - 5$

$2 \,|\, 3 \quad 1 \quad -2 \quad 0 \quad -5$
$\quad 6 \quad 14 \quad 24 \quad 48$
$3 \quad 7 \quad 12 \quad 24 \quad 43$

$f(2) =$ $(2, 43)$

Ex: If $f(x) = 3x^5 + 6x^4 - 75x + 10$, $f(-3)$ using synthetic sub.

$-3 \,|\, 3 \quad 6 \quad 0 \quad 0 \quad -75 \quad 10$
$\quad -9 \quad 9 \quad -27 \quad 81 \quad -18$
$3 \quad -3 \quad 9 \quad -27 \quad 6 \quad -8$

$(-3, -8)$

What is the remainder when $P(x) = x^{15} + 3x^{10} + 2$ is divided by $x - 1$.

$P(+1) = 1^{15} + 3(1)^{10} + 2 = 6$

Remainder is 6

Factor Theorem: the binomial x – a is a factor of the polynomial

$$f(x) \Leftrightarrow f(a) = 0$$

Ex: Is x – 2 a factor of $f(x) = x^4 - 4x^3 + 5x^2 + 4x - 12$

Find $f(2)$, if it equals zero, then x – 2 is a factor!

Ex: Factor $f(x) = 3x^3 + 14x^2 - 28x - 24$ given that x – 2 is a factor. $(x-2)(3x+2)(x+6)$

$$\begin{array}{r|rrrr} 2 & 3 & 14 & -28 & -24 \\ & & 6 & 40 & 24 \\ \hline & 3 & 20 & 12 & \boxed{0} \end{array}$$

$3x^2+20x+12$

$(3x+2)(x+6)$

zeros $x=2$, $x=-2/3$, $x=-6$

Ex: x = -4 x = -1 x = 2

M.K.

$f(x) = (x + 4)(x + 1)(x - 2)$

Ex: If x + 2 is a factor of $x^3 - x^2 - 10x - 8$. Find the others.

$$\begin{array}{r|rrrr} -2 & 1 & -1 & -10 & -8 \\ & & -2 & 6 & 8 \\ \hline & 1 & -3 & -4 & 0 \end{array}$$

x^2-3x-4

$(x-4)(x+1)(x+2)$

Roots and Zeros:

Ex: Given that $-\frac{3}{2}$ is a root, solve $12x^3 + 6x^2 - 20x - 3 = 0$

$$\begin{array}{r|rrrr} -3/2 & 12 & 6 & -20 & -3 \\ & & -18 & 18 & 3 \\ \hline & 12 & -12 & -2 & \boxed{0} \end{array}$$

Ex: $w^3 + w^2 - 6w - 120$

$12x^2-12x-2$

The real zeros will be the x-intercepts of the graph. $2(6x^2-6x-1)$

The roots will be 5, $-3 + i\sqrt{15}$, $-3 - i\sqrt{15}$

Q.F.

One real and two imaginary

Write the equation with roots at $\frac{2\pm\sqrt{3}}{5}$ and 1

Remember they belong to the set of Complex #'s.

$\left(x-\frac{2+\sqrt{3}}{5}\right)\left(x-\frac{2-\sqrt{3}}{5}\right)$ $(25x^2-20x+1)(x-1)$

$\frac{2+\sqrt{3}}{5}+\frac{2-\sqrt{3}}{5}=\frac{4}{5}=\frac{-b}{a}=\frac{20}{25}$ $\frac{2+\sqrt{3}}{5}\cdot\frac{2-\sqrt{3}}{5}=\frac{1}{25}=\frac{c}{a}$

Ex: Given that -2 is a zero of $f(x) = x^3 - 2x^2 + 5x + 26$, find all other zeros.

$$\begin{array}{r|rrrr} -2 & 1 & -2 & 5 & 26 \\ & & -2 & 8 & -26 \\ \hline & 1 & -4 & 13 & 0 \end{array} \qquad x = \frac{4 \pm \sqrt{16-4(13)}}{2} = \frac{4 \pm 6i}{2} = 2 \pm 3i$$

$x^2 - 4x + 13$

$x = -2 \quad x = 2 \pm 3i$

Ex: Solve $p(x) = x^3 + x + 10$, if -2 is a root.

$$\begin{array}{r|rrrr} -2 & 1 & 0 & 1 & 10 \\ & & -2 & 4 & -10 \\ \hline & 1 & -2 & 5 & 0 \end{array} \qquad x = \frac{2 \pm \sqrt{4-4(5)}}{2} \quad x = \frac{2 \pm 4i}{2} = 1 \pm 2i$$

$x^2 - 2x + 5$

$x = -2 \quad x = 1 \pm 2i$

Fundamental Theorem of Algebra: Every polynomial equation with degree greater than zero has at least one root in the set of complex numbers(includes real numbers).

Therefore, a polynomial equation of the form $P(x) = 0$ of degree n, has exactly n roots.

# of solutions = degree of the polynomial

☆ **Every polynomial of ODD degree (with real coefficients) has at least one REAL root** ☆

Ex. $x^3 + 4x^2 + 4x = 0$ has 3 solutions

Ex. $x^4 - 10x^2 + 9 = 0$ has 4 solutions

***imaginary roots occur in CONJUGATE PAIRS (if polynomial coefficients are real)**

The multiplication of complex conjugates gives a real number.

$$(a + bi)(a - bi) = a^2 + b^2$$

$$(a - bi)(a + bi) = a^2 - b^2$$

If 2 + 3i is a root, then so is 2 – 3i

**Multiplicity: how many times a solution repeats.

Ex. $x^2(x - 4)(x - 4)(x - 4) = 0$

Solutions: {0(mult. of 2), 4(mult. of 3)}