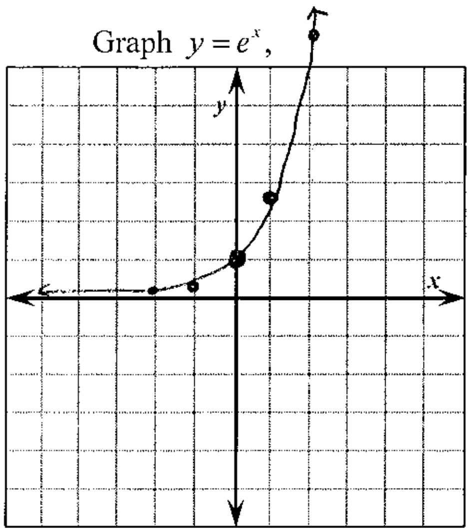
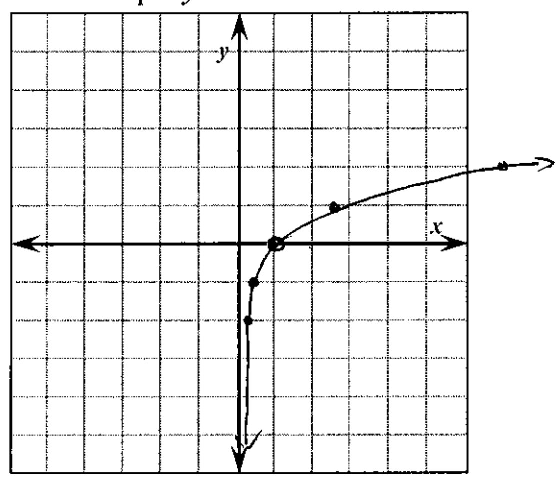
Exponential Function:
Has the form $y = a^x$, where $a > 0$, $a \neq 1$ and x is any real number.

Graph $y = e^x$,



Graph $y = \ln x$



The Natural Base *e* (Euler's number):

An irrational number, symbolized by the letter *e*, appears as the base in many applied exponential functions. This irrational number is approximately equal to 2.72. More accurately, $e = 2.71828$.. The number *e* is called the natural base.

The Natural Logarithm function has the following properties:

1. The domain is $(0, \infty)$ and the range is $(-\infty, \infty)$

2. The function is continuous, increasing, and one-to-one.

3. The graph is concave downward.

Logarithm Properties: If a and b positive numbers and n is rational, then the following are true.

1. $\ln(1) = 0$ $\ln(e) = 1$

2. $\ln(ab) = \ln a + \ln b$

3. $\ln(a^n) = n \ln a$

4. $\ln\left(\dfrac{a}{b}\right) = \ln a - \ln b$

Definition of the Natural Logarithm Function: $\ln x = \int_1^x \frac{1}{t} dt, \; x > 0$

Expanding Logarithms:

Ex: $\ln\frac{3x^2}{5} =$ $\ln 3 + \ln x^2 - \ln 5$

$\ln 3 + 2\ln x - \ln 5$

Ex: $\ln\frac{(2x+1)^3}{\sqrt{x^2-1}} =$ $3\ln(2x+1) - \frac{1}{2}\ln(x^2-1)$

Derivative of the Natural Logarithmic Function

$$\frac{d}{dx}[\ln x] = \frac{1}{x},\ x > 0 \qquad\qquad \frac{d}{dx}[\ln u] = \frac{1}{u}\frac{du}{dx} = \frac{u'}{u},\ u > 0$$

Find the derivative of each of the following:

Ex: $y = \ln(5x)$

$u = 5x \quad du = 5$

$y' = \frac{5}{5x} = \frac{1}{x}$

Ex: $y = \ln|\sin x|$

$u = \sin x$

$du = \cos x$

$y' = \frac{\cos x}{\sin x} = \cot x$

Ex: $y = 3x\ln(x)$

$y' = 3x \cdot \frac{1}{x} + 3 \cdot \ln x$

$y' = 3 + 3\ln x$

Ex: $y = \ln\sqrt{x-3}$

$y = \frac{1}{2}\ln(x-3)$

$y' = \frac{1}{2}\left(\frac{1}{x-3}\right) = \frac{1}{2(x-3)}$

Ex: $y = \ln\frac{x(x^2-5)}{\sqrt{x+6}}$

$y = \ln x + \ln(x^2-5) - \frac{1}{2}\ln(x+6)$

$y' = \frac{1}{x} + \frac{2x}{x^2-5} - \frac{1}{2}\frac{1}{x+6}$

Ex: $y = \ln(\ln x)$

$u = \ln x$

$du = 1/x$

$y' = \frac{\frac{1}{x}}{\ln x} = \frac{1}{x\ln x}$

Logarithmic Differentiation:

Ex: $y = \frac{(x-2)^2}{\sqrt{x^2+4}}$

$\ln y = \ln\left(\frac{(x-2)^2}{\sqrt{x^2+4}}\right)$

$\frac{y'}{y} = 2\ln(x-2) - \frac{1}{2}\ln(x^2+4)$

$\frac{y'}{y} = 2\left(\frac{1}{x-2}\right) - \frac{1}{2}\left(\frac{2x}{x^2+4}\right)$

$\frac{y'}{y} = \frac{2}{x-2} - \frac{x}{x^2+4}$

$\frac{y'}{y} = \frac{2x^2+4-x^2+2x}{(x-2)(x^2+4)} = \frac{x^2+2x+4}{\sim}$

$y' = \left[\frac{(x-2)^2}{\sqrt{x^2+4}} \cdot \frac{x^2+2x+4}{(x-2)(x^2+4)}\right] = \frac{(x-2)(x^2+2x+4)}{x^2+4\sqrt{x^2+4}}$

Ex: Find all relative extrema and inflection points for $y = \frac{x^2}{2} - \ln x$

$x > 0$

$y' = x - \frac{1}{x}$

0 1

$y'' = 1 + x^{-2}$

$x = 0$ not in domain