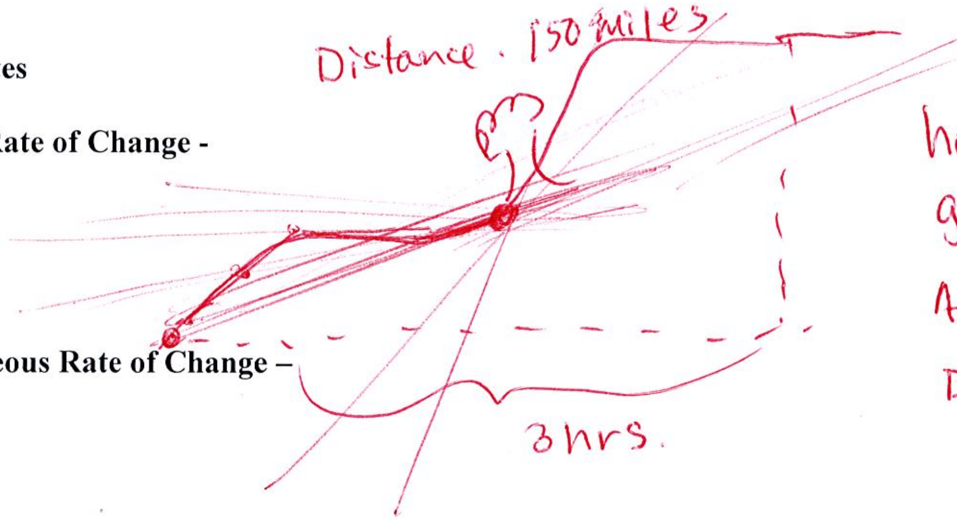
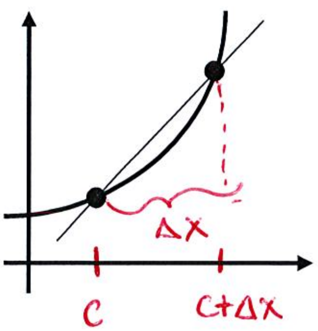
Unit 3 Notes

Average Rate of Change -



how long ?
gas money ?
Arrival time.
Departure time.

Instantaneous Rate of Change –



You can approximate the slope by using the secant line through the point of tangency and a second point on the curve. If $(c, f(c))$ is the point of tangency and $(c + \Delta x, f(c + \Delta x))$ is a second point on the graph of f.

Slope $= \dfrac{f(c+\Delta x) - f(c)}{c+\Delta x - c} = \dfrac{f(c+\Delta x) - f(c)}{\Delta x}$

secant.

The right hand side of this equation is the difference quotient. The denominator Δx is the change in x, and the numerator $\Delta y = f(c + \Delta x - f(c))$ is the change in y.

Approximating the slope:

tangent line.

$\Delta x \to 0$

Definition of Tangent Line with Slope *m*

other examples

If f is defined on an open interval containing c, and if the limit $\lim\limits_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x} = \lim\limits_{\Delta x \to 0} \dfrac{f(c + \Delta x) - f(c)}{\Delta x} = m$

exists, then the line passing through $(c, f(c))$ with slope m is the tangent line to the graph of f at the point $(c, f(c))$.

Ex: Find the slope of the tangent line of $f(x) = x^2$ at x = -2, x = 0, x = 2.

$$\lim_{\Delta x \to 0} = \frac{f(-2+\Delta x)^2 - f(-2)}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{4 - 4\Delta x + \Delta x^2 - 4}{\Delta x} = -4$$

$$\frac{f(0+\Delta x) - f(0)}{\Delta x}$$

$$\frac{\Delta x^2}{\Delta x} = 0$$

*** Notice that the slope varies with x's when the function is nonlinear.**

$$\lim_{\Delta x \to 0} \frac{f(2+\Delta x) - f(2)}{\Delta x}$$

$$= 4$$

$x^{1/2} = \frac{1}{2}(x)^{-1/2} = \frac{1}{2\sqrt{x}} = \frac{1}{4}$

Ex: Find the slope of the tangent line of $f(x) = \sqrt{x}$ at x = 4, x = 9.

$$\lim_{\Delta x \to 0} \frac{f(4+\Delta x) - f(4)}{\Delta x} = \frac{\sqrt{4+\Delta x} - 2}{\Delta x} \cdot \frac{\sqrt{4+\Delta x} + 2}{\sqrt{4+\Delta x} + 2} = \frac{4+\Delta x - 4}{}$$

$$= \frac{1}{\sqrt{4+\Delta x} + 2} = \frac{1}{4}$$

$x = 9 = 1/6$

The Derivative of a Function: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

tangent line! $y - 2 = \frac{1}{4}(x-4)$

Norm $y - 2 = -4(x-4)$

provided the limit exists. For all x for which this limit exists, f' is a function of x.

Other notation for the derivative: $\frac{dy}{dx}$ "*the derivative of y with respect to x*", y' $D_x[y]$, $\frac{d}{dx}[f(x)]$

Ex: Find the derivative of $f(x) = 3x - 1$

$$\lim_{h \to 0} \frac{3(x+h) - 1 - [3x-1]}{h}$$

$$\frac{3x + 3h - 1 - 3x + 1}{h}$$

$$\frac{3h}{h} = 3$$

Ex: Find the derivative of $f(x) = x^3 + 2x$

$$\lim_{h \to 0} (x+h)^3 + 2(x+h) - [x^3 + 2x]$$

$$\frac{x^3 + 3x^2h + 3xh^2 + h^3 + 2x + 2h - x^3 - 2x}{h}$$

$$= 3x + 2$$

Alternative Definition of a Derivative: $f'(x) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$

This is useful to show the relationship between differentiability and continuity.

This requires that both one-sided limits exist.

Ex: Write the equation of the tangent line and normal line of $f(x) = \frac{1}{x}$ at x = 3 $(3, \frac{1}{3})$

$$\lim_{x \to 3} \frac{\frac{1}{x} - \frac{1}{3}}{x - 3} = \frac{-1}{3x}$$

$$y - \frac{1}{3} = \frac{-1}{9}(x-3)$$

Differentiability – what does this mean? When is a function not differentiable?

derivative = slope of tangent line.
So if not differential ⇒ no slope of tan line
no tangent line or discontinuity.

Discont
Jumps
Holes

Ex: $f(x) = |x + 3|$

cusp

(abrupt change in slope

Ex: $f(x) = \sqrt[3]{x}$

show

not at $x = 0$

vertical tangent line

Ex: $f(x) = \frac{1}{x}$

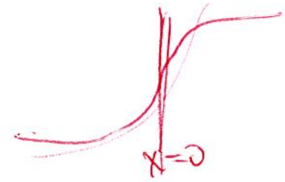
not at $x = 0$

If f is differentiable at x = c, then f is continuous at x = c.

* Differentiability Implies Continuity, not other way.

Vertical Tangents: $x^{1/3}$ $\quad x=0$

$$\lim_{x\to 0}\frac{f(x)-f(0)}{x-0}$$



$$=\lim_{x\to 0}\frac{1}{x^{2/3}}=\frac{1}{\sqrt[3]{x^2}}\quad \infty$$

If f is continuous $\quad \lim_{x\to c^+}\frac{f(c+\Delta x)-f(c)}{\Delta x}=\infty$

$\lim_{x\to c^-}$ " $=-\infty$