**Parametric Equation:**
We have been doing graphs in x and y and now we will introduce a third variable to represent a curve in the plane. This third variable is called a parameter and often represents "time" though it could mean other things.

Ex: We could graph the curve representing a baseball that is hit at a 45° angle at a velocity of 50ft per sec.

To introduce t, we will write both x and y as a function of t and get parametric equations

Definitions
Plane curve- the set of ordered pairs $(f(t), g(t))$ if $f$ and $g$ are continuous functions of t

Parametric equations- $x = f(t)$ and $y = g(t)$ Parameter is t!

Sketching a plane curve
Choose increasing values of t and make an x, y, t table by substituting t into the equation.

Ex: $x = t^2 - 9$ $-3 \le t \le 3$

$y = \frac{t^2}{3}$

| t | -3 | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| x | 0 | -5 | -8 | -9 | -8 | -5 | 0 |
| y | 3 | 4/3 | 1/3 | 0 | 1/3 | 4/3 | 3 |

$x = t^2 - 9$ $y = \frac{t^2}{3}$

$x + 9 = t^2$ $y = \frac{x+9}{3}$

$y = \frac{1}{3}x + 3$

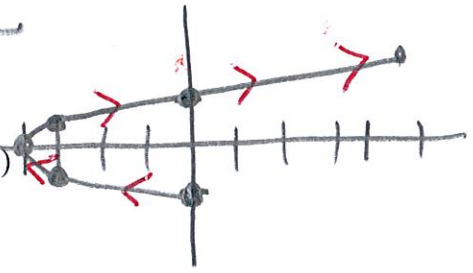
Two different sets of parametric equations can have the same graph.

Ex: $x = t^2 - 4$

$y = \frac{t}{2}$ $-2 \le t \le 3$

Graph each

| t | -2 | -1 | 0 | 1 | 2 | 3 |
|---|---|---|---|---|---|---|
| x | 0 | -3 | -4 | -3 | 0 | 5 |
| y | -1 | -1/2 | 0 | 1/2 | +1 | 3/2 |



**Eliminating the parameter:**

Converting from parametric equations to rectangular equations (x,y)
1. Solve for t in one equation
2. Substitute what you get into the other equation

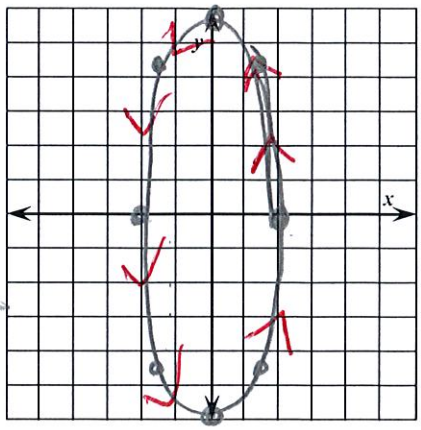
Ex: $x = t^2 - 9$ $x + 9 = t^2$

$y = \frac{t^2}{3}$ $y = \frac{x+9}{3} = \frac{1}{3}x + 3$

Ex: sometimes the parameter represents an angle rather than time.
Sketch the curve represented by eliminating the parameter

$x = 2\cos\theta$ $\quad 0 \le \theta \le 2\pi$
$y = 6\sin\theta$

| $\theta$ | 0 | $\pi/4$ | $\pi/2$ | $3\pi/4$ | $\pi$ | $5\pi/4$ | $3\pi/2$ | $7\pi/4$ | $2\pi$ |
|---|---|---|---|---|---|---|---|---|---|
| $x$ | 2 | $\sqrt{2}$ | 0 | $-\sqrt{2}$ | $-2$ | $-\sqrt{2}$ | 0 | $\sqrt{2}$ | 2 |
| $y$ | 0 | $3\sqrt{2}$ | 6 | $3\sqrt{2}$ | 0 | $-3\sqrt{2}$ | $-6$ | $-3\sqrt{2}$ | 0 |




Rectangular equation- good for sketching curve

Parametric equations- good for seeing position, direction, and speed $\frac{x}{2} = \cos\theta \quad \frac{y}{6} = \sin\theta$

$\cos^2\theta + \sin^2\theta = 1$

Examples:

Ex: $x = t$
$y = \frac{1}{2}t$ $\quad y = \frac{1}{2}x$

$t = x + 1$

Ex: $x = t - 1$
$y = \frac{t}{t-1}$

$\frac{x^2}{4} + \frac{y^2}{36} = 1$

$y = \frac{x+1}{x+1-1} = \frac{x+1}{x}$

Ex: $x = \cos\theta$
$y = 2\sin 2\theta$

Finding parametric equations for a given graph
You can let t be anything

Ex: $y = x^2 - 4$

$\begin{cases} x = t \\ y = t^2 - 4 \end{cases}$ $\quad x = \pm\sqrt{t+4}$ $\quad y = t$ $\quad x = t + 25$ $\quad y = (t+25)^2 - 4$

Ex: $\frac{x^2}{4} - \frac{y^2}{16} = 1$

$y = \pm\sqrt{4t^2 - 16}$
$x = t$

$16\left(\frac{t^2}{4} - \frac{y^2}{16} = 1\right)$

$4t^2 - y^2 = 16$
$4t^2 - 16 = y^2$

Ex: $x = a\cos t \quad y = a\sin t \quad a > 0$ is a constant

Ex: $x = \frac{3t^2}{4}$ $\quad -4 \le t \le 4$
$y = t$

Ex: $x = 3t^2 + 12t + 12$ $\quad -4 \le t \le 0$
$y = 2t + 4$