

Factoring- express a polynomial as the product of its prime factors.

1. G.C.F.
2. Binomial – Look for a pattern
3. Trinomial – UNFOIL (Guess & Check)
4. 4 terms - Grouping

Ex: $5x^2 - 20$

$$5(x+2)(x-2)$$

Ex: $x^4 - 64$

$$(x+2\sqrt{2})(x-2\sqrt{2})(x^2+8)$$

Ex: $3\cos^3\theta - 3\cos\theta$

$$3\cos\theta(\cos^2\theta - 1) \text{ or } 3\cos\theta(-\sin^2\theta)$$

Ex: $16x^5 - 250x^2$

$$2x^2(2x-5)(4x^2+10x+25)$$

Ex: $\cot^2\theta + \tan^2\theta$

irreducible
(prime)

Ex: $6x^2 + 15x + 9$

$$3(2x+3)(x+1)$$

Ex: $2x^5 - 12x^3 + 18x$

$$2x(x^2-3)(x^2+3)$$

Ex: $5x^3 + 15x^2 - 270$

$$5x(x+9)(x-6)$$

Ex: $125x^3 + 27y^6$

$$(5x+3y^2)(25x^2-15xy^2+9y^4)$$

or $2x(x+\sqrt{3})(x-\sqrt{3})(x^2+3)$

Ex: $x^3 - 3x^2 + 4x - 12$

$$(x^2-4)(x-3) \\ (x+2)(x-2)(x-3)$$

Ex: $e^{-x} - xe^{-1} + \frac{2x^2}{e^x}$

$$e^{-1}(1 - xe^{x-1} + 2x^2)$$

Ex: $2\sqrt{x} + 6x^{\frac{3}{2}}$

$$2x^{\frac{1}{2}}(1 + 3x)$$

Ex: $x^{\frac{1}{2}} - 15x^{\frac{1}{4}} + 50$

$$(x^{\frac{1}{4}} - 10)(x^{\frac{1}{4}} - 5)$$

Ex: $\frac{x^2}{16} - \frac{7x}{2} + 49$

$$\left(\frac{x}{4} - 7\right)\left(\frac{x}{4} - 7\right)$$

Ex: $5(x+y)^2 + 10(x+y) + 5$

$$5[(x+y)+1][(x+y)+1]$$

Ex: $x^4 - 8x^3 + 24x^2 - 32x + 16$

$$(x-2)^4$$

Ex: $\sec^2\theta + 23\sec\theta + 120$

$$(\sec\theta + 15)(\sec\theta + 8)$$

Ex: $x^3 + 2x^2 + 3x + 6$

$$(x^2+3)(x+2)$$

Ex: $2x^3 + 5x + 7 = (x+1)(2x^2 - 2x + 7)$

Ex: $x^3 - 4x^2 + 2x = (x-1)(x^2 - 3x - 1)$

Notes:

Ex: $(2x+1)^{\frac{3}{2}} x^{\frac{1}{2}} + (2x+1)^{\frac{5}{2}} x^{-\frac{1}{2}}$

$(2x+1)^{\frac{3}{2}} x^{\frac{1}{2}} \left[1 + (2x+1)^{-1} \right]$

Ex: $(x-3)^3(x^2 + 4x + 4) - 2(x^2 - 6x + 9)(x+2)^2$

$(x-3)^2(x+2)^2 [(x-3) - 2]$

or $(x-3)^2(x+2)^2 [x-5]$

Simplifying Complex Fractions and using Conjugates:

Rules for dealing with fractions are stated $\frac{a}{b}$ so we must get them in that form.

Ex: $\frac{\frac{2}{x+2}}{\frac{1}{x+2} + \frac{2}{x}}$

Ex: $\frac{\frac{x}{x-3} - \frac{2}{3}}{\frac{10}{3x} + \frac{x^2}{x-3}}$

Ex: $\frac{\frac{2x}{y} + 1 - \frac{y}{x}}{\frac{2x}{y} + \frac{y}{x} - 3}$

$\frac{2x}{3x+4}$

$\frac{x^2 + 3x}{3x^3 + 10x - 30}$

$\frac{x+y}{x-y}$

Simplifying and Rationalizing:

Ex: $\frac{3x+9}{6x}$ (3)

$\frac{x+3}{2}$

Ex: $\frac{x^{\frac{1}{2}} - x^{\frac{1}{3}}}{x^{\frac{1}{6}}}$ ($x^{\frac{1}{6}}$)

$x^{\frac{1}{3}} - x^{\frac{1}{6}}$

Ex: $\frac{(x+1)^3(x-2) + 3(x+1)^2}{(x+1)^4}$

$\frac{(x+1)(x-2) + 3}{x+1}$ or $\frac{x^2 - x + 1}{x+1}$

Ex: $\frac{1 - (\sin(x) + \cos(x))^2}{2 \sin(x)}$

$-\cos \theta$

Ex: $\frac{\sin x}{\tan x}$

$\cos \theta$

Ex: $\frac{3-4i}{2+5i}$

$\frac{-14-23i}{29}$

Free up things.

Ex: $\frac{\sqrt{x+1}-1}{x}$

$\frac{1}{(\sqrt{x+1} + 1)}$

Ex: $\frac{4-\sqrt{x}}{x-16}$

$\frac{4-\sqrt{x}}{(\sqrt{x}-4)(\sqrt{x}+4)} = \frac{-1}{\sqrt{x}+4}$

Ex: $\frac{\sqrt{x+5}-\sqrt{5}}{x}$

$\frac{1}{\sqrt{x+5} + \sqrt{5}}$