12.1 Arithmetic Sequences & Series

1. A **sequence** is an ordered list of numbers. Each number in the list is called a **term** of the sequence. The first term of a sequence is denoted as a_1. The second term is denoted as a_2. The term in the nth position is called the nth term and is denoted as a_n. The term before a_n is a_{n-1}.

   A sequence is a function whose <u>range</u> is the <u>terms</u> of the sequence and the <u>domain</u> is the <u>position</u> of each term.

2. **Finding the Next Term:** To find the next term in an arithmetic sequence, first find the common difference, then add the common difference to the next term.

3. Find the next three terms.

   a. $-12, -1, 10, \dots$ b. 7, 10, 13, ... c. $r-4, r-1, r+2, \dots$

4. **Arithmetic sequences:**
   The sequence $a_1, a_2, a_3, a_4, \dots a_n$ is arithmetic if there is a number d such that:

   $$a_2 - a_1 = d$$
   $$a_3 - a_2 = d$$

   Where d is the common difference.

   Ex: 6, 9, 12, 15, ….3n + 3 The common difference is 3 because 9 – 6 = 3. $d = 3$

   Ex: 2, -3, -8, -13, …, -5n + 7 The common difference is –5 because -3–2 = -5. $d = -5$

5. **<u>Explicit Formula:</u>** a formula that defines the nth term. $\boxed{a_n = dn + c}$

   We will look at c as being the a_0 **(a sub not) term**. Think y-intercept!

   The book will use $a_n = a_1 + (n-1)d$ or $a_{n+1} = a_n + d$.

6. **Recursive Sequence:** is a sequence in which each term is defined using the previous terms.
   Each Arithmetic Sequence can be written recursively using $a_{n+1} = a_n + d$

Ex: Find a formula of an arithmetic sequence whose common difference is 4 and whose first term is 3.

$a_n = dn + c$ We know $d = 4$. $a_1 = 3$. So $a_0 = 3 - 4$. $a_0 = -1$

Recursive Will Be
$a_{n+1} = a_n + 4$

$a_n = 4n - 1$. The terms of this sequence are: 3, 7, 11, 15, …, 4n – 1.

Ex: Find the formula of the arithmetic sequence whose first term is 3 and whose second term is –1.

$a_n = dn + c$ We know $a_1 = 3$ and $a_2 = -1$. So $d = -4$. a_0 must be 3–(-4) = 7

$a_n = -4n + 7$ The terms of this sequence are: 3, -1, -5, -9, …, -4n + 7.

List the terms $a_0 = 7$

7. Find the 35th term in the sequence 11, 4, -3, … $d = -7$ $a_0 = 18$

$a_n = -7n + 18$

$a_{35} = -7(35) + 18$

$a_{35} = -227$

8. Find the 20th term in the sequence for which $a_1 = -27$ and $d = 3$.

List a few

a_0 $-27, -24, -21, -18$ $a_n = 3n - 30$

$a_0 = -30$ $a_{20} = 3(20) - 30 \Rightarrow 30$

9. Find the first term in the sequence for which $a_4 = 229$ and $d = 8$.

205 $\underline{213}$ $\underline{221}$ $\underline{229}$

In General,

$a_4 = 8(4) + a_0$

$229 = 32 + a_0$

$197 = a_0$

$205 = a_1$

10. The fifth term of an arithmetic sequence is 25 and the 12th term is 60. Write the first several terms of this sequence.

$a_5 = 25$ $a_{12} = 60$

$a_{12} = a_5 + 7d$ (where 7 is the difference in the term numbers).

$60 = 25 + 7d$ $35 = 7d$ $d = 5$

Explicit is $a_n = 5n + 0$

Since $a_5 = 25$ we can subtract 5 to get each term in the sequence down to the first.

5, 10, 15, 20, 25

11. **Arithmetic means: the terms between any two nonconsecutive terms of an arithmetic sequence.**

The terms between 2 given terms of an arithmetic sequence are called <u>*arithmetic means*</u>.

$10, \underbrace{13, 16, 19}_{\text{3 arithmetic means}}, 22$

$10, \underbrace{14, 18}_{\text{2 arithmetic means}}, 22$

12. Form an arithmetic sequence that has five arithmetic means between -11 and 19.

-11 __, __, __, __, __, 19 $a_n = 5n + a_0$

$6d = 19 - -11$ $a_n = 5n - 16$

$6d = 30$ $d = 5$

13. Form an arithmetic sequence that has six arithmetic means between -12 and 23.

-12 __ __ __ __ __ __ 23 $a_n = 5n - 17$

$7d = 23 - -12$ $d = 5$

CLASS EXAMPLE! -3 __ __ __ -19

$a_n = -4n + 1$

14. **Summation Notation:** the sum of a sequence is also known as an **Arithmetic Series.**

End

$$\sum_{k=1}^{m} c_k = c_1 + c_2 + c_3 + \ldots + c_m$$

Start

15. **Sigma Notation**: the sum of the first n terms of a sequence (called a series)

Ex: $\sum_{i=2}^{5}(2i) = 28$ Ex: $\sum_{k=3}^{8}(2k+3) = 84$ Ex: $\sum_{j=5}^{10}(3-j) = -27$

$2(2)+2(3)+2(4)+2(5)$

$2(3)+3 + 2(4)+3 + 2(5)+3 + 2(6)+3 + 2(7)+3 + 2(8)+3$

$(3-5)+(3-6)+(3-7)+(3-8)+(3-9)+(3-10)$

16. **The Sum of an Arithmetic Series:** $S_n = \frac{n}{2}(a_1 + a_n)$ This means that we add the first and last terms, then multiply by the number of terms divided by 2.

Ex: Find the sum of the integers from 1 to 500.

$S_n = \frac{n}{2}(a_1 + a_n)$ n = 500, a_1 = 1 and a_n = 500 $S_n = \frac{500}{2}(1+500) = 250(501) = 125,250$

17. Find the sum of the first 27 terms in the series -14, -8, -2, ……+ 142.

$S_n = \frac{27}{2}(-14+142) = 1,728$

18. Find the sum of the first 32 terms in the series -12, -6, 0, …. $d = 6$ $a_0 = -18$

$a_{32} = ?$ Use what we learned earlier

$a_n = 6n - 18$ $a_{32} = 174$

$S_n = \frac{32}{2}(-12+174) = 2,592$

19. Find n for a series for which $a_1 = 5$, $d = 3$, and $S_n = 440$.

$a_n = 3n+2$

$440 = \frac{n}{2}(5 + a_n)$

$440 = \frac{n}{2}(5+3n+2)$

$880 = n(3n+7)$

$3n^2+7n-880=0$

$(3n+55)(n-16)$

$n=16$

20. Nimisha starts a college savings account for her daughter on her sixth birthday. She plans to deposit $25 the first month and then increase the deposit by $5 each month. How much will she have deposited in twelve years?

25 30 35 40 45

12 years = 144 months

$a_n = 5n+20$

$a_{144} = 740$

$S_n = \frac{144}{2}(25+740) =$

$S_n = \$55,080$

21. The number of seats in the first row is 20, the second row is 23, the third row is 26, and so on. How many seats are in Row 16? How many seats is there altogether in those 16 rows?

20 23 26

$a_n = 3n+17$

Row 16 $= 3(16)+17 = 65$

Total $= \frac{16}{2}(20+65) = 680$ seats