## 5.1 Angles & Their Measures

**An angle is determined by rotating array at its endpoint.**

Starting side is **initial** – ending side is **terminal**
Endpoint of ray is the vertex of angle.
Origin = vertex
**Standard Position:** When an angle is at the origin and its initial side lies along the positive x-axis.

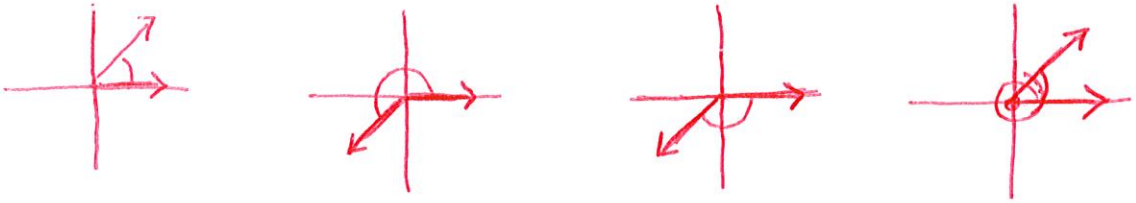
**Positive angles: counter-clockwise** **Negative angles: clockwise**

**Quadrantal Angle:** An angle whose terminal side lies on the x-axis or the y-axis

Measurement of angle is amount of rotation from initial side to terminal side.

**Draw each angle in standard position:**

a) $45°$ *angle* b) $225°$ *angle* c) $-135°$ *angle* c. d) $405°$ *angle*



**Radians:** One radian is the measure of a central angle $\theta$ that intercepts an arc equal in length to the radius of the circle.

Just over 6 radians in a full circle hence $\mathbf{2\pi}$
**Make sure to make clear $\pi$ in radians is 180 degrees and $\pi$ as a distance is 3.14…..**

Because the radian measure of an angle of one full revolution is $2\pi$ you obtain.

$\frac{1}{2}$ revolution $\frac{2\pi}{2} = \pi$ radians = 180 degrees

$\frac{1}{6}$ revolution $\frac{2\pi}{6} = \frac{\pi}{3}$ radians = 60 degrees

**Degrees:** – 1 degree is equivalent to a rotation of $\frac{1}{360}$ a revolution about the vertex.

**360 degrees = $2\pi$ radians**

All angles can be broken down into minutes and seconds where 60 minutes equal one degree and 60 seconds equals one minute. This helps to get a more precise angle measure.

You can find this under the **ANGLE** button on your calculator (DMS-degrees-minutes-seconds)

**How to convert from Degrees to Radians:** radians = 1 degree = $\frac{\pi}{180}$

**How to convert from Radians to Degrees:** degrees = 1 radian = $\frac{180}{\pi}$ degrees

Convert to Radians:

a. 45° $45 \cdot \frac{\pi}{180}$ $\pi/4$

b. 150° $150 \cdot \frac{\pi}{180}$ $5\pi/6$

c. 72° $72 \frac{\pi}{180}$ $2\pi/5$

d. 270° $270 \cdot \frac{\pi}{180}$ $3\pi/2$

e. 99° $99 \cdot \frac{\pi}{180}$ $11\pi/20$

Convert to Degrees:

a. $\frac{\pi}{2} \cdot \frac{180}{\pi}$ $90^0$

b. $\frac{3\pi}{4}$ $135^0$

c. $\frac{2\pi}{5}$ $72^0$

d. $\frac{5\pi}{6}$ $150^0$

e. $\frac{3\pi}{14} \cdot \frac{180}{\pi}$ $\frac{270}{7}{}^0$

**Coterminal:** An angle of $x°$ is coterminal with angles of $x° + k \cdot 360°$, where k is an integer. Coterminal angles have the same initial sides and terminal sides.

Ex: 0 and $2\pi$ are coterminal

Ex: $\frac{\pi}{6}$ and $\frac{13\pi}{6}$ are coterminal.

Determine two coterminal angles for each:

a. 165° $165 + 360 = 525^0$ $165 - 360 = -195$

b. 420° $420 - 360 = 60^0$ $60 - 360 = -300$

c. -120° $-120 + 360 = 240^0$ $240 + 360 = 600^0$

d. – 135° $225^0, -495^0$

a) positive angles you will subtract $2\pi$

b) negative angles you will add $2\pi$ $\frac{13\pi}{6} - 2\pi = \frac{\pi}{6}$

**Reference angle**: is the acute angle $\theta$ formed by the terminal side of $\theta$ and the horizontal axis. What are the reference angles?

1) -45 degrees $45^0$

2) 200 degrees $20^0$

3) 123 degrees $57^0$

4) -400 degrees $40^0$

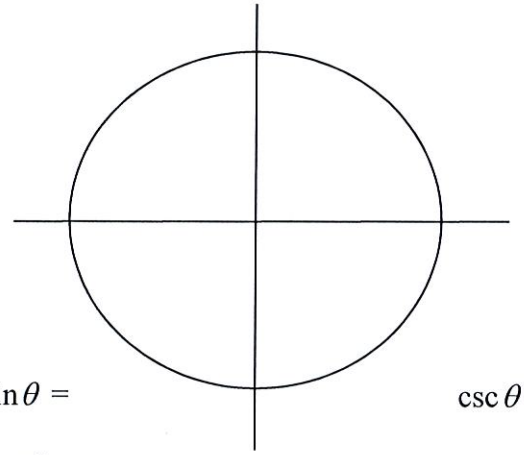
5) $\frac{2\pi}{5}$ $2\pi/5$

6) $\frac{25\pi}{12}$ $\pi/12$

7) $\frac{-3\pi}{4}$ $\pi/4$

8) $\frac{-12\pi}{5}$ $2\pi/5$

Unit Circle:



$\sin\theta =$ $\csc\theta =$

$\cos\theta =$ $\sec\theta =$

$\tan\theta =$ $\cot\theta =$

Notice since $\sin\theta = y$ and $\cos\theta = x$

**$x^2 + y^2 = 1$ so $\sin^2\theta + \cos^2\theta = 1$ is an identity**

**Notice since $\sin\theta = y$ that sine is positive in quadrant one and two.**

**Also $\cos\theta$ is = x and that is positive in quadrant one and four.**

**Since tangent is = $\frac{\sin\theta}{\cos\theta}$ it is positive in one and three.**