11.1 Rational Exponents

1. **Properties of Exponents:**
Suppose m and n are positive integers, and a and b are real numbers.
Then the following properties hold.

Product Property: $a^m a^n = a^{m+n}$
Power of a Power Property: $(a^m)^n = a^{mn}$
Power of a Quotient Property: $(\frac{a}{b})^m = \frac{a^m}{b^m}$
Power of a Product Property: $(ab)^m = a^m b^m$
Quotient Property: $\frac{a^m}{a^n} = a^{m-n}$, where $a \neq 0$.

2. Evaluate each expression.

a. $3^2 3^3$ $3^5 = 243$

b. $(3^2)^3$ $3^6 = 729$

c. $\left(\frac{1}{2}\right)^3$ $\frac{1}{2}^3 = \frac{1}{8}$

d. $(2a)^3$ $8a^3$

e. $\frac{2^5}{2^3}$ $2^2 = 4$

3. **Negative Exponents:**

To get rid of a negative exponent, "move" the base that is being carried to the negative exponent.

Ex. $2^{-1} = \frac{2^{-1}}{1} = \frac{1}{2^1}$ or $\frac{3}{b^{-2}} = \frac{3b^2}{1}$

4. **Definition of $b^{\frac{1}{n}}$:**

For any real number $b \geq 0$ and any integer n > 1, $b^{\frac{1}{n}} = \sqrt[n]{b}$
This also holds when b<0 and n is odd.

5. Evaluate:

a. $625^{\frac{1}{4}}$ $= 5$

b. $3^{\frac{1}{2}} \cdot 21^{\frac{1}{2}}$ $\sqrt{3} \cdot \sqrt{21} = \sqrt{63} = 3\sqrt{7}$

c. $8^{-\frac{2}{3}}$ $= \frac{1}{8^{2/3}} = \frac{1}{\sqrt[3]{8^2}} = \frac{1}{4}$

6. **Definition of Rational Exponents:**

For any nonzero number b, and any integers m and n with n > 1, $b^{\frac{m}{n}} = \sqrt[n]{b^m} = (\sqrt[n]{b})^m$
Except when $\sqrt[n]{b}$ is not a real number.

7. Evaluate each.

a. $81^{\frac{5}{4}}$ b. $36^{\frac{3}{2}}$ or $(6^2)^{3/2}$ c. $64^{\frac{5}{3}}$

✓ $\sqrt[4]{81^5}$ or $(3^4)^{5/4} = 3^5 = 243$ $\quad \sqrt[2]{36^3} = 6^3 = 216$ $\quad (4^3)^{5/3} = 4^5 = 1024$

6·6 6·6·6·6

8. Express using rational exponents.

a. $\sqrt{23}$ b. $\sqrt[3]{63}$ c. $\sqrt[4]{16z^2}$ d. $\sqrt[3]{5x^2y}$ e. $\sqrt[4]{27x^4y^3}$

$23^{1/2}$ $\quad 63^{1/3}$ $\quad 2(z^{1/2})$ $\quad 5^{1/3}x^{2/3}y^{1/3}$ $\quad x(27^{1/4}y^{3/4})$

9. Express using radicals.

a. $6^{\frac{1}{5}}$ b. $4^{\frac{1}{3}}$ c. $c^{\frac{2}{5}}$ d. $(x^2)^{\frac{4}{3}}$ e. $(5a)^{\frac{2}{3}}b^{\frac{5}{3}}$

$\sqrt[5]{6}$ $\quad \sqrt[3]{4}$ $\quad \sqrt[5]{c^2}$ $\quad x^{8/3} = x^2 \cdot x^{2/3}$ $\quad 5^{2/3}a^{2/3} \cdot b^1 \cdot b^{2/3}$

$x^2 \cdot \sqrt[3]{x^2}$ $\quad b\sqrt[3]{25a^2b^2}$

10. Simplify each.

a. $y^{\frac{5}{3}}y^{\frac{7}{3}}$ b. $(b^{\frac{1}{3}})^{\frac{3}{5}}$ c. $\sqrt[3]{a^4b^8}$ d. $\sqrt{8m^5n^4}$ e. $\sqrt[4]{32a^9b^{11}}$

$y^{12/3} = y^4$ $\quad b^{1/5}$ or $\sqrt[5]{b}$ $\quad a^{4/3}b^{8/3} =$ $\quad 2m^2n^2\sqrt{2m}$ $\quad 2a^2b^2\sqrt[4]{2ab^3}$

$ab^2\sqrt[3]{ab^2}$ or $2m^2n^2(2m)^{1/2}$

<u>11.2 Exponential Functions</u>

11. Evaluate to the nearest ten thousandth.

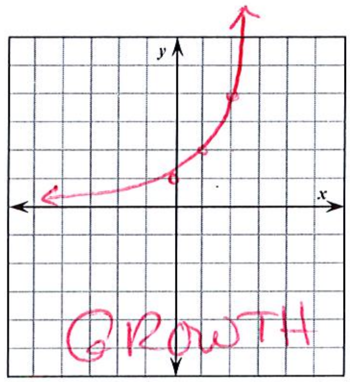
a. $5^{\sqrt{2}}$ b. 0.4^{π}

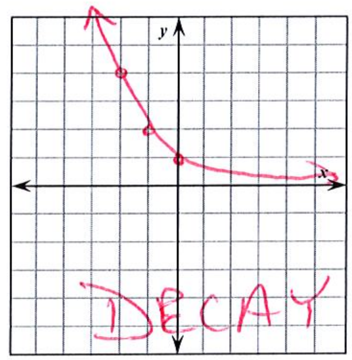
9.7385 $\quad$ .05621

12. Exponential Function:

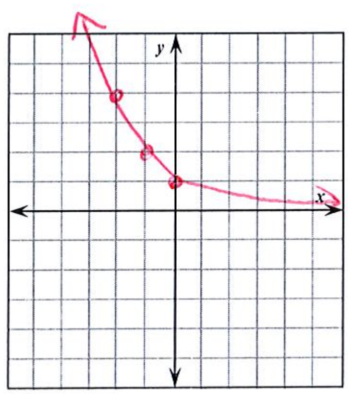
Has the form $y = a^x$, where a is a positive real number. Ex. 2^x, 3^x, $\left(\frac{1}{5}\right)^x$

13. Graph and compare the graphs of $y = 2^x$ and $y = 2^{-x}$.



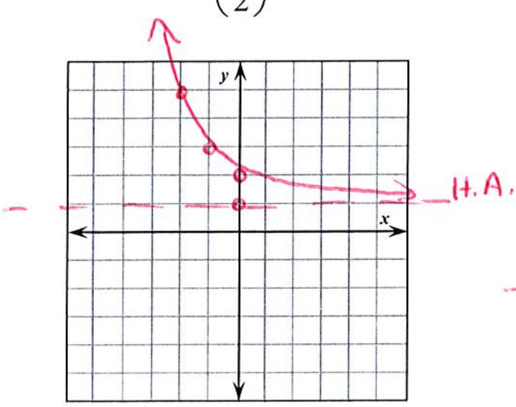


14. Graph $y = \left(\frac{1}{2}\right)^x$.

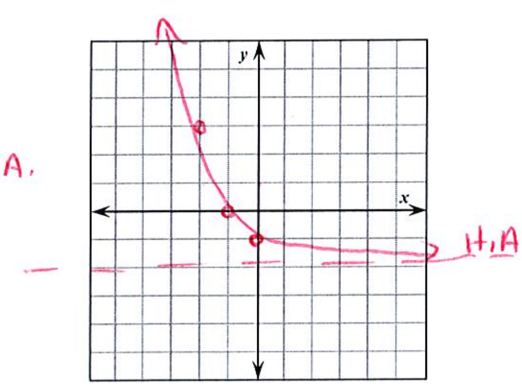


Describe the transformations of the parent graph $y = \left(\frac{1}{2}\right)^x$.

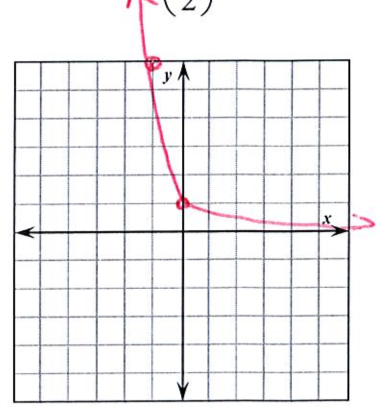
$y = \left(\frac{1}{2}\right)^x + 1$



$y = \left(\frac{1}{2}\right)^x - 2$



$y = 3\left(\frac{1}{2}\right)^x$



Homework: Page 602 #1-51 odd and Page 612 #17-28 and Worksheet

Every other Odd