11.4 Logarithmic Functions

1. Definition of a Logarithmic Function:

The logarithmic function $y = \log_a x$, where a > 0 and $a \neq 1$, is the inverse of the exponential function $y = a^x$.

2. Write in exponential form. $b^P = \# \Leftrightarrow \log_b n = P$

a. $\log_5 125 = 3$ $5^3 = 125$

b. $\log_{13} 169 = 2$ $13^2 = 169$

c. $\log_4 \frac{1}{4} = -1$ $4^{-1} = 1/4$

d. $\log_{\frac{1}{5}} 25 = -2$ $(1/5)^{-2} = 25$

e. $\log 100 = 2$ $\uparrow$ common logarithm

f. $\log \frac{1}{1000} = -3$ $\log_{10} .001 = -3$

3. Write in logarithmic form.

a. $8^3 = 512$ $\log_8 512 = 3$

$\log_x 16 = 2$ $x = 4$ $\cancel{x = -4}$

b. $3^3 = 27$ $\log_3 27 = 3$

c. $5^{-3} = \frac{1}{125}$ $\log_5 \frac{1}{125} = -3$

4. Evaluate each expression.

a. $\log_2 16 = x$

$2^x = 16$

$2^x = 2^4$

$x = 4$

b. $\log_{12} 144$

$12^x = 144$

$x = 2$

c. $\log_{16} 4 = x$

$16^x = 4^1$

$(4^2)^x = 4^1$

$2x = 1 \Rightarrow x = 1/2$

d. $\log_3 243$ $x = 5$

e. $\log_2 \frac{1}{32}$ $x = -5$

f. $\log_3 \frac{1}{81}$ $x = -4$

5. Properties of Logarithms:

Suppose m and n are positive numbers, b is a positive number other than 1, and p is any real number. Then the following properties hold.

> Product Property: $\log_b mn = \log_b m + \log_b n$
>
> Quotient Property: $\log_b \frac{m}{n} = \log_b m - \log_b n$
>
> Power Property: $\log_b m^p = p \log_b m$
>
> Property of Equality: If $\log_b m = \log_b n$, then $m = n$.

6. Write the expression as the logarithm of a single quantity.

a. $\log x + 3\log y$

$\log x + \log y^3$

$\log xy^3$

b. $\log(2x+5) - \log x$

$\log \frac{2x+5}{x}$

c. $\frac{1}{2}(\log_5 x + \log_5 y) - 2\log_5(x+1)$

$\frac{1}{2}\log_5 xy - \log_5 (x+1)^2$

$\log_5 \frac{\sqrt{xy}}{(x+1)^2} = \log_5 \frac{\sqrt{xy}}{x^2+2x+1}$

7. Use the properties of logarithms to write the expression as a sum, difference, and/or constant multiple of logarithms. (Assume all variables are positive).

a. $\log_9 9x$ — $\log_9 9 + \log_9 x = \boxed{1 + \log_9 x}$

b. $\log_4\left(\frac{64}{y}\right)$ — $\log_4 64 - \log_4 y$ $\boxed{3 - \log_4 y}$

c. $\log_b\left(\frac{x^2 y}{z^2}\right)$ — $\log_b x^2 + \log_b y - \log_b z^2$ $\boxed{(2\log_b x + \log_b y) - 2\log_b z}$

8. Evaluate each.

a. $\log_3(\log_4 64)$ — $\log_4 64 = x$ — $\log_3(3) = 1$

b. $\log_2[\log_2(\log_3 81)]$ — $\log_2(\log_2 4)$ — $\log_2 2 = 1$

Doesn't Always Equal 1!

9. If $\log_7 6 = x$ and $\log_7 5 = y$, express the following in terms of x and y:

a. $\log_7 35 = \log_7 7 + \log_7 5$ — $1 + y$

b. $\log_7 7.2 = \log_7 \frac{36}{5} = \log_7 36 - \log_7 5$ — $\log_7 6 + \log_7 6 - \log_7 5$ — $2x - y$

10. Inverse Properties of Logarithms:

For $b > 0$,

$\log_b b^x = x$ The logarithm with base b of b raised to a power equals that power.

$b^{\log_b x} = x$ b raised to the logarithm with base b of a number equals that number.

11. Evaluate each.

a. $\log_4 4^x = x$

b. $\log_7 7^8 = 8$

c. $6^{\log_6 9} = 9$

d. $3^{\log_3 17} = 17$

$8^{\log_2 5} = (2^3)^{\log_2 5} = 2^{3\log_2 5} = 5^3$

12. Solve each of the following.

a. $\log_b 5 = -\frac{1}{3}$ $= \frac{1}{125}$

$b^{-1/3} = 5$ $\quad \frac{1}{b^{1/3}} = 5$

$\frac{1}{\sqrt[3]{b}} = 5$ $\quad 5\sqrt[3]{b} = 1$ $\quad \sqrt[3]{b} = 1/5$

b. $\log_3 5 + \log_3 x = \log_3 10$ — 2

c. $\log 16 - \log 2t = \log 2$ — 4

d. $\log(2x+5) = \log(5x-4)$

$2x + 5 = 5x - 4$

$9 = 3x$

$3 = x$

e. $\log_3(4x+5) - \log_3(3-2x) = 2$

$\log_3 \frac{4x+5}{3-2x} = 2$

$\frac{9}{1} = \frac{4x+5}{3-2x}$

$27 - 18x = 4x + 5$

$22 = 22x$

$1 = x$

f. $2\log 6 - \frac{1}{3}\log 27 = \log x$ — $x = 12$

$2a^2 + 4 = 36$

g. $\log_6(a^2+2) + \log_6 2 = 2$ — $a = \pm 4$

h. $\log_3(x-5) + \log_3(x+3) = 2$ — $x^2 - 2x - 15 = 9$ — $x = 6$

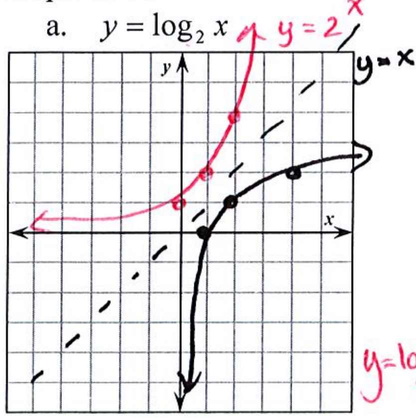
$(x-6)(x+4) = 0$

$x = \cancel{-4}$

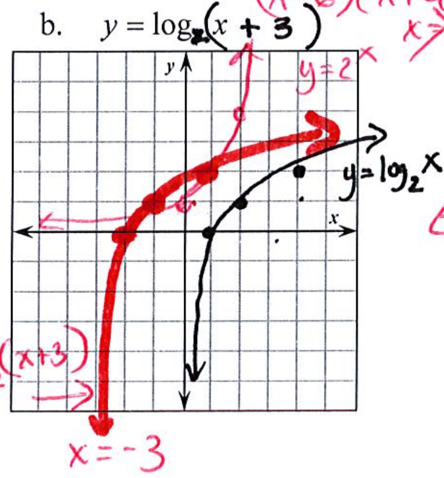
i. $\log x + \log(x-3) = 1$ — $x^2 - 3x - 10 = 0$ — $x = 5$

13. Graph each.

a. $y = \log_2 x$ — $y = 2^x$



b. $y = \log_2(x+3)$ — $y = 2^x$



c. $y = \log x$ — Graph 10^x