14.1 Part II Permutations and 14.2 Permutations with Repetitions & Circular Permutations Notes

1. **Permutation:** A permutation of n differenct elements is an ordering of the elements such that one element is first, one is second, one is third, and so on. **ORDER MATTERS!!**

2. Permutation is an ordered arrangement of items that occurs when
   a. No item is used more than once.
   b. The order of arrangement makes a difference

Ex: There are 10 finalists in a figure skating competition. How many ways can gold, silver, and bronze medals be awarded?

$10 \cdot 9 \cdot 8$ This is $\frac{10!}{7!} = 720$

Ex: You and 19 friends have decided to form an Internet marketing consulting firm. The group needs to choose three officers-a CEO, an operating manager, and a treasurer. In how many ways can those offices be filled?

$20 \cdot 19 \cdot 18 = {}_{20}P_3$ or $\frac{20!}{17!} = 6,840$

3. **Permutation Formula:** ${}_nP_r = \frac{n!}{(n-r)!}$ or $P(n,r) = \frac{n!}{(n-r)!}$

4. Eight people enter the Best Pie contest. How many ways can blue, red, and green ribbons be awarded?

${}_8P_3 = \frac{8!}{5!} = 8 \cdot 7 \cdot 6 = 336$

5. Suppose you want to rearrange the letters of the word ALGEBRA to see if you can make a different arrangement. If the two A's were not identical, the seven letters in the word could be arranged in P(7,7) or 7! ways. But since the A's are identical, what are we going to do?

divide out repeats → $\frac{7!}{2!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 2,520$

6. **Permutations with Repetitions**:

Find the number arrangements for the word:

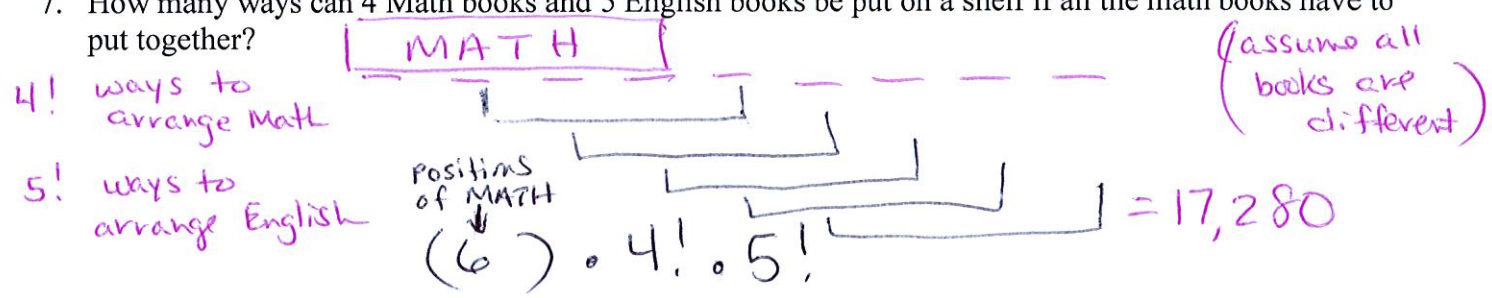
$\frac{11!}{4!4!2!}$ = 34,650

A. BANANA $\frac{6!}{3!2!} = 60$ (the a's, the n's)

B. BASEBALL $\frac{8!}{2!2!2!} = 5040$

C. MISSISSIPPI

7. How many ways can 4 Math books and 5 English books be put on a shelf if all the math books have to put together?



8. How many ways can 4 Math books and 5 English books be put on a shelf if all the math books and the English books have to be put together?

$(2) \cdot 5! \cdot 4! = 5{,}760$

MATH/ENG OR ENG/MATH

9. **Circular Permutations:** If n objects are arranged in a circle, then there are $\frac{n!}{n}$ or $(n-1)!$ permutations of the n objects around the circle; they do not have a beginning or an end.

$\frac{5!}{5}$ can rotate this arrangement 5 times.

10. On the buffet there are 7 different appetizers from which to choose. The appetizers are arranged on a revolving tray. How many ways can the appetizers be organized?

$\frac{7!}{7}$ or $6! = 720$

11. How many ways can 8 people be seated at a square table?

Even though the table is square, people can rotate without changing who sits next to who

$7! = 5040$

12. How many ways can 5 men and 5 women be seated at a round table if they have to alternate the men and women?

$5! \cdot 5!$ (men, women)

$= \frac{5! \cdot 5!}{10} = 1{,}440$ (can rotate 10 spots)

If n objects on a circle are arranged in relation to a fixed point, then there are $n!$ Permutations.

Any reference makes the arrangement linear

If the arrangement can be physically turned over or flipped over, the reflection of the arrangement is possible, divide by 2. (For Example, a key ring can be flipped over but a football team in a huddle cannot)

13. How many ways can 7 beads be placed on a bracelet with no clasp?

$\frac{6!}{2} = 6 \cdot 5 \cdot 4 \cdot 3 = 360$ (no reference)

14. How many ways can 7 beads be placed on a bracelet that has a clasp?

$\frac{7!}{2} = 2{,}520$ (reference)

Homework: Finish Permutation Worksheet from class, pg. 755 #5 – 34, 37 – 40, 46