review ICMAB 2016 integration

Name: ______________________ **Date:** ______________

1. If $F(x) = \int_0^{x^2+1} f(t)\,dt$ and $f(10) = 2$, then $F'(3) =$

$A(x^2+1) - A(0)$

A. -30 B. 30 C. 45 D. 60

$F'(x) = f(x^2+1) \cdot 2x - f(0)$

$= 12$

2. Find the definite integral $\int_0^3 2x(x^3 - 2x^2 + 5)\,dx$.

$2x^4 - 4x^3 + 10x$

A. $\frac{5}{306}$ B. $\frac{306}{5}$ C. $\frac{4}{207}$ D. 61

$\frac{2}{5}x^5 - x^4 + 5x^2 \Big|_0^3$

3. $\int \frac{1}{\sqrt{2x+1}}\,dx =$

$(2x+1)^{-1/2} = 2(2x+1)^{1/2}$

A. $\sqrt{x^2+x} + C$ B. $\frac{1}{\sqrt{x^2+x}} + C$

C. $\sqrt{2x+1} + C$ D. $\frac{1}{2\sqrt{x^2+x}} + C$

$u = 2x+1$
$du = 2\,dx$

$\frac{1}{2}\int u^{-1/2}$

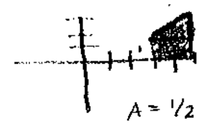
$\frac{1}{2} \cdot 2\,u^{1/2}$

4. If $F'(x) = f(x)$ for all x, and if f is a continuous function, then $\int_1^6 f(5x)\,dx =$ _____ by using u substitution.

A. $5(F(6) - F(1))$ B. $5(F(30) - F(5))$

C. $\frac{1}{5}(F(6) - F(1))$ D. $\frac{1}{5}(F(30) - F(5))$

5. Evaluate: $\int_3^5 |x - 2|\,dx$



$A = 1/2\,(2)(1+3)$

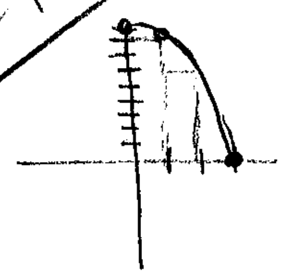
A. 0 B. 4 C. 5 D. 6

6. Find the lower sum for the region bounded by $f(x) = 9 - x^2$ and the x-axis between $x = 0$ and $x = 3$.

A. $9 - \frac{27}{2n} + \frac{27}{6n^2}$ B. $18 + \frac{27}{2n} - \frac{27}{6n^2}$

C. $18 - \frac{27}{2n} - \frac{27}{6n^2}$ D. $18 - \frac{27}{2n} + \frac{27}{6n^2}$

SKIP



7. Integrate: $\int \frac{\cos^3\theta}{2 - 2\sin^2\theta}\,d\theta$

$\frac{\cos^3\theta}{2(1-\sin^2\theta)}$ $\frac{\cos\theta}{2}$

A. $\frac{1}{2}\sin\theta + C$

B. $\frac{\cos^3\theta}{8(\theta - \frac{1}{3}\sin^3\theta)} + C$

C. $\frac{3}{4}\cot^2\theta + C$

D. $\frac{1}{2}\ln\left|1 - 2\sin^2\theta\right| + C$

8. Determine: $\int (2 - \frac{1}{x})x^{-3}\,dx$ $\frac{2}{x^3} - \frac{1}{x^4}$

A. $\frac{1}{3x^3} - \frac{1}{x^2} + C$ B. $-\frac{1}{x^3} + \frac{1}{x^2} + C$

C. $3x^{-3} - x^{-2} + C$ D. $-3x^{-3} - x^{-2} + C$

$\int 2x^{-3} - x^{-4}$

$-x^{-2} + \frac{1}{3}x^{-3} + C$

9. Evaluate the indefinite integral: $\int \frac{x}{(x+2)^{2/3}}\,dx$

A. $\frac{3}{4}(x+2)^{2/3} - 6(x+2)^{1/6} + C$

B. $\frac{3}{4}(x+2)^{4/3} - 6(x+2)^{1/3} + C$

C. $3(x+2)^{1/3} + C$

D. $\frac{3}{4}\ln(x+2)^{3/2} + C$

$u = x+2$ $x = u-2$
$du = 1\,dx$

$\int (u-2)\,u^{-2/3} = \int u^{1/3} - 2u^{-2/3}$

$3/4\,u^{4/3} - 6u^{1/3} + C$

10. Evaluate: $\frac{d}{dx}\int_3^x (3t^4+7)^3\,dt$

$A(x) - A(3)$

A. $(3x^4+7)^3$ B. $(x^4+7x)^2$

C. 15625000 D. 0

11. An object moves such that its acceleration $a(t) = 2(1-3t)$. Given $v(0) = -5$ and $x(0) = 4$, find $x(1)$.

A. 1 B. 2 C. −1 D. 5

$a(t) = -6t + 2$

$v(t) = -3t^2 + 2t - 5$

$x(t) = -t^3 + t^2 - 5t + 4$

$x(1) = -1$

12. Use $a(t) = -10\,\text{ft/s}^2$ as the acceleration due to gravity on the planet Mathematica. A rock is thrown vertically upward from the ground with an initial velocity of 40 feet per second. How high will the rock go?

A. 120 feet B. 100 feet

C. 40 feet D. 80 feet

$v(t) = -10t + 40$

$s(t) = -5t^2 + 40t$

$-5t(t-8)$

13. Use the Fundamental Theorem of Calculus to evaluate $\int_{-2}^{2} (\sqrt[3]{t} - 10)\,dt$. $t^{1/3} - 10$

A. −40 B. $-\frac{5}{3}$ C. 0 D. $\frac{81}{2}$

$\frac{3}{4}t^{4/3} - 10t \Big|_{-2}^{2}$

$\frac{3}{4}(2)^{4/3} - 20 - \left[\frac{3}{4}(-2)^{4/3} - (20)\right]$

14. Which of the following is the indefinite integral for $\int \sin^3(3x)\cos(3x)\,dx$?

$u = \sin(3x)$
$du = \cos(3x)\cdot 3$

A. $\frac{1}{8}\sin^4(3x)\cos^2(3x) + C$

B. $\frac{1}{4}\sin^4(3x) + C$

C. $\frac{1}{12}\sin^4(3x) + C$

D. $\frac{1}{48}\sin^4(3x)\cos^2(3x) + C$

$\frac{1}{3}\int u^3\,du$

$\frac{1}{3}\cdot\frac{1}{4}u^4$

15. On the planet Euclid, acceleration due to gravity is $16\,\text{m/sec}^2$. An object is fired vertically upward from ground level so that it takes 8 seconds before it returns to the ground.

a) What is the maximum height the object reaches? 128

b) What was the initial velocity with which the object was fired? 64 m/sec

$a(t) = -16$

$v(t) = -16t + 64$

$s(t) = -8t^2 + 64t$

$-8t(t-8)$

16. Use u substitution to find $\int x\sqrt{1-x}\,dx =$.

$u = 1-x$

$du = -1$

$x = -u+1$

A. $\dfrac{2-3x}{2\sqrt{1-x}} + C$

B. $\dfrac{x^2}{3}(1-x)^{3/2} + C$

C. $-\dfrac{2}{3}(1-x)^{3/2} + \dfrac{2}{5}(1-x)^{5/2} + C$

D. $\dfrac{2}{3}(1-x)^{3/2} + \dfrac{2}{5}(1-x)^{5/2} + C$

$-\int(-u+1)\,u^{1/2}$

$-\int -u^{3/2} + u^{1/2}$

$-\left[-\tfrac{2}{5}u^{5/2} + \tfrac{2}{3}u^{3/2}\right]$

17. Perform the indicated operation: $\int (3x^2 - 2x + 5)\,dx$

A. $x^3 - x^2 + 5x + C$ B. $\frac{1}{3}x^3 - \frac{1}{2}x^2 + 5x + C$

C. $x^3 - 2x^2 + C$ D. $6x - 2 + C$

$x^3 - x^2 + 5x + C$

18. Use the Mean Value Theorem for Integrals to find at least one value of c for $f(x) = x^2 - 4x - 3$ on the closed interval $[1,3]$. Then use that value of c to find the average value of $f(x)$ on $[1,3]$.

A. $-\dfrac{40}{3}$ B. $-\dfrac{20}{3}$ C. $-\dfrac{10}{6}$ D. -6

$\frac{1}{2}\int x^2 - 4x - 3$

$\left[\frac{1}{3}x^3 - 2x^2 - 3x\right]_1^3$ $-22/3$

$-18 - {-14/3}$ $(3x\quad)(x\quad)$

$x^2 - 4x - 3 = -22/3$

$3x^2 - 12x\ \ 13\ \ \ 22$

19. A golf cart is driven along path. Its velocity $v(t)$ is represented by the formula $2 + \sin\frac{t}{\pi}$. Which of the following is a formula for its position $x(t)$, if $x(0) = 0$?

A. $2t - \pi\cos\frac{t}{\pi} + \pi$ B. $t - \pi\cos\frac{t}{\pi} + \pi$

C. $2t + \pi\cos\frac{t}{\pi} + \pi$ D. $2t + \cos\frac{t}{\pi} + \pi$

$\int 2 + \int \sin(u)$ $u = \frac{t}{\pi}$

$\int 2 + \pi\int \sin(\frac{t}{\pi})$ $du = 1/\pi\,dt$

$2t - \pi\cos(\frac{t}{\pi}) + C$

$C = \pi$

20. Integrate: $\int \sin 3x\,dx$

A. $\frac{1}{3}\cos 3x + C$ B. $-\frac{1}{3}\cos 3x + C$

C. $\frac{1}{3}\sin 3x + C$ D. $\frac{1}{3}\sin 3x \cos 3x + C$

$u = 3x\ dx$

$du = 3\,dx$

21. $\dfrac{d}{dx}\displaystyle\int_x^2 \frac{5t}{2t^3-3}\,dt =$

A. $\dfrac{5x}{2x^3-3}$ B. $-\dfrac{5t}{2t^3-3}$

C. $-\dfrac{5x}{2x^3-3}$ D. $\dfrac{10}{13}$

22. Evaluate: $\int_1^2 (x^2 + 2x)\,dx$

A. $\frac{3}{8}$ B. $\frac{3}{4}$ C. $\frac{16}{3}$ D. $\frac{20}{3}$

$\frac{1}{3}x^3 + x^2 \Big|_1^2$

$8/3 + 4 - [1/3 + 1]$

23. Integrate: $\int \frac{1}{x^4}\,dx$ x^{-4} $-3x^{-3}$

A. $-\frac{1}{3x^3}$ B. $\frac{5}{x^5}$

C. $-\frac{1}{5x^5} + C$ D. $-\frac{1}{3x^3} + C$

24. Given $g(x) = A + h(x)$ and $\int_1^5 h(x)\,dx = A$, find the average value of $g(x)$ over the interval $[1, 5]$ in terms of A.

A. $\frac{A}{4}$ B. $\frac{5A}{4}$ C. $4A$ D. $3A$

$\frac{1}{4}\int A + h(x)$

$\frac{1}{4}\int_1^5 A + \int_1^5 h(x)$

$4A + A$ $\frac{5A}{4}$

25. Find the definite integral $\int_{-1}^2 (2x - 1)\,dx$.

A. -4 B. -2 C. 4 D. 0

$x^2 - x \Big|_{-1}^2$

$2 -$

26. $\frac{d}{dx}\int_{4x}^{x^3} \sqrt{t+3} =$ $3x\sqrt{x^3+3} - 4\sqrt{4x+3}$

$A(x^3) - A(4x)$

$\sqrt{x^3+3} \cdot 3x^2 - \sqrt{4x+3} \cdot 4$

27. Find the particular solution of the function defined by the following properties:

$f''(x) = 18x$; $f'(1) = 5$; and $f(3) = 80$. Find $f(x)$.

A. $3x^3 - 4x$ B. $12x^3 - 4x + 11$

C. $18x^3 - 5x + 11$ D. $3x^3 - 4x + 11$

$f'(x) = 9x^2 - 4$

$f(x) = 3x^3 - 4x + 11$

28. Assume f is differentiable everywhere, where $x \in \mathbb{R}$, and

I. $f(1) = -1$ $f(x) = \frac{2}{3}ax^3 - 2bx^2 + C$

II. $f'(1) = -10$

III. $f'(x) = 2ax^2 - 4bx$ $4ax - 4b$

IV. $f''(1) = -4$ $4a - 4b = -4$

$-2a \pm 4b = \pm 10$

Find $f(x)$. $2a = 6$

$a = 3$

A. $2x^3 - 8x^2 + 5$ B. $2x^3 - 8x^2$ $b = 4$

~~C.~~ $x^3 - 8x^2 + 5$ D. $2x^3 - 8x^2 + 1$

A. $6x^2 - 16x$ B. $6x^2 - 16x$

C. $3x^2 - 16x$ D. $6x^2 - 16x$

$f'(x) = 6x^2 - 16x + 0$

$f(x) = 2x^3 - 8x^2 + 5$

29. A function is defined by the following properties:

$f'(x) = 6x$ and $f(1) = -1$. Find $f(x)$.

A. $3x^2 - 4$ B. $6x^2 - 1$

C. $6x - 7$ D. $6x - 2$

$f(x) = 3x^2 - 4$

30. Use the Fundamental Theorem of Calculus to evaluate $\int_1^4 \sqrt{x}\,dx$. $x^{1/2}$

A. 1   B. $-\frac{14}{3}$ C. $\frac{14}{3}$ D. -1

$\frac{2}{3}x^{3/2} \Big|_1^4$

31. Find the position function, $P(t)$, given $a(t) = -6$, $v(3) = -10$, and $P(2) = 44$. $a(t)$ is the acceleration and $v(t)$ is the velocity.

$v(t) = -6t + 8$

$P(t) = -3t^2 + 8t + 40$

A. $P(t) = -3t^2 - 40t + 8$

B. $P(t) = -3t^2 - 8t - 40$

C. $P(t) = -3t^2 + 8t - 40$

D. $P(t) = -3t^2 + 8t + 40$

32. Find the indefinite integral: $\int \frac{x}{\sqrt{x-1}}\,dx$

A. $\frac{2}{3}(x-1)^{3/2} + 2\sqrt{x-1} + C$ $u = x - 1$

B. $\frac{1}{2}x^2\sqrt{x-1} + C$ $u - 1 = x$

C. $\frac{3x+1}{4\sqrt{x-1}} + C$ $\int (u-1)\,u^{-1/2}$

D. $x\sqrt{x-1} + C$ $\int u^{1/2} - u^{-1/2}$

$\frac{2}{3}u^{3/2} - 2u^{1/2} + C$

33. Find the indefinite integral: $\int \frac{3 + 4x^{3/2}}{\sqrt{x}}\,dx$

A. $\frac{3}{2}\sqrt{x} + 2x^2 + C$ B. $\frac{3}{2}x^{-3/2} + 2x^2 + C$

C. $6\sqrt{x} + 2x^2 + C$ D. $3x^{-1/2} + 4x + C$

$\int \frac{3}{\sqrt{x}} + \int 4x$

$6x^{1/2} + 2x^2 + C$

34. Given $f(x) = Ax^3 + Bx^2 + cx + D$, and

I. $f(1) = -1$ $f' = 3Ax^2 + 2Bx + C$

II. $f'(1) = 10$ $6Ax + 2B$

III. $f''(0) = -4$ $2B = -4$

IV. $f''(1) = 14$ $B = -2$

$6A - 4 = 14$

What is the value of $(A + B + C + D)$? $A = 3$

A. 3   B. 10   C. 1   D. -1

$3A + 2B + C = 10$

$A + B + C + D = -1$

$A = 3 \quad B = -2 \quad C = 5 \quad D = -7$

35. $\frac{d}{dx}\int_1^{\sin x}(1+t^2)\,dt =$ $A(\sin x) - A(1)$

A. $(1+2\sin x)\cos x$ B. $(1+t^2)\cos x$

C. $(1+\sin^2 x)$ D. $(1+\sin^2 x)\cos x$

$1+\sin^2 x \cdot \cos x$

36. Evaluate: $\int \frac{2x^2+3x^{1/2}+4}{x^{1/2}}\,dx$ $\int 2x^{3/2} + 3 + \frac{4}{x^{1/2}}$

A. $3x+\frac{4}{5}x^2\sqrt{x}+4+C$

B. $\frac{3}{2}x+\frac{4}{5}x^2\sqrt{x}+4+C$

C. $3x+\frac{5}{4}x^2\sqrt{x}+2+C$

D. $3x+\frac{4}{5}x^2\sqrt{2x}+4+C$

$\frac{4}{5}x^{5/2} + 3x + 8x^{1/2}$

37. Use u substitution to find the indefinite integral for $\int \sin\frac{x}{2}\,dx$.

A. $\cos\frac{x}{2}+C$ B. $-2\cos\frac{x}{2}+C$

C. $2\sin^2\frac{x}{2}+C$ D. $-\frac{1}{2}\cos\frac{x}{2}+C$

$u = \frac{1}{2}x$

$du = \frac{1}{2}dx$

$2\int \sin u$

38. $\int \sin x \sec^2 x\,dx =$ $\int \tan x \cdot \sec x$

A. $\cos x + C$ B. $\sec x + C$

C. $\sin^{-1}x + C$ D. $\sin x\cos x + C$

39. A mouse in a maze has a constant acceleration of 8 and at time $t = 2$ its velocity is 9. When time $t = 1$, it passed the starting point. Find $P(t)$, the function describing its distance from the starting point at any given time t.

A. $P(t) = 4t^2 - 7t + 3$ B. $P(t) = 4t^2 - 7t - 3$

C. $P(t) = t^2 - 7t + 3$ D. $P(t) = t^2 + 7t + 3$

$a(t) = 8$

$v(t) = 8t - 7$

$s(t) = 4t^2 - 7t$

40. Given the function $f(x) = 2x^2 + 3$, find its average value on the interval $[0, 2]$.

A. $\frac{17}{3}$ B. $\frac{34}{3}$ C. 4 D. 27

$\frac{1}{2}\int_0^2 2x^2 + 3$

$\frac{1}{2}\left[\frac{2}{3}x^3 + 3x\right|_0^2$

$\frac{1}{2}\left(\frac{16}{3} + 6\right)$

41. Integrate: $\int 5\sec x\tan x\,dx$

A. $5\sec^3 x\tan x + C$

B. $5\sec x + C$

C. $5\left[\sec^3 x + \sec x\tan^2 x\right] + C$

D. $5\sec^2 x + C$

42. $\dfrac{d}{dx}\displaystyle\int_5^{x^3} \frac{dt}{t-7} =$

A. $\dfrac{1}{x-7}$ B. $\dfrac{3x^2}{x-7}$

C. $\dfrac{3x^2}{x^3-7}$ D. $\dfrac{x^3}{x^3-7}$

43. $\displaystyle\int x\sqrt{8-4x^2}\,dx =$

A. $\frac{2}{3}(8-4x^2)^{3/2}+C$

B. $-\frac{1}{8}(8-4x^2)^{3/2}+C$

C. $-\frac{1}{12}(8-4x^2)^{3/2}+C$

D. $\frac{1}{4}(8-4x^2)^{3/2}+C$

$u = 8 - 4x^2$

$du = -8x\,dx$

$-\frac{1}{8}\int u^{1/2}$

$-\frac{1}{8} \cdot \frac{2}{3} u^{3/2}$

44. $\displaystyle\int \frac{\cos^3 x - \sin^2 x}{\cos^2 x}\,dx =$

A. $\sin x - \sec x + C$ B. $\sin x - \tan x + x + C$

C. $\sin x - \dfrac{\tan^3 x}{3} + C$ D. $\frac{3}{4}\cos x - \tan^3 x + C$

$\int \cos x - \int \tan^2 x$

$-\int \sec^2 x - 1$

45. Find the average value of $2x$ over the interval $a \le x \le b$.

A. $b^2 - a^2$ B. $2b^2 - a^2$

C. $a - b$ D. $b + a$

$\frac{1}{b-a}\int_a^b 2x$

x^2

$\frac{b^2-a^2}{b-a}$

$\frac{(b-a)(b+a)}{b-a}$

46. Consider the integral $\int x^2\,dx$ from $x = 2$ to $x = 6$. Using a Riemann sum with 4 sub-intervals calculate the area under the curve, and above the x-axis, using *midpoints*.

A. 72 B. 66 C. 69 D. 58

47. Choose the correct statement given that $\int_0^5 f(x)\,dx = 7$ and $\int_2^5 f(x)\,dx = -1$.

A. $\int_0^2 f(x)\,dx = 6$ B. $\int_5^2 f(x)\,dx = -1$

C. $\int_2^0 f(x)\,dx = 8$ D. $\int_0^2 f(x)\,dx = 8$

48. If $\int_a^b f(x) = 4A - 5B$, then $\int_a^b (f(x) - 6)\,dx =$

A. $2A - B$ B. $-2A - 11B$

C. $10A - 11B$ D. $4A - 5B - 6$

$\int_a^b f(x) - \int_a^b 6$

$6x$

$4A - 5B - [6b - 6A]$

49. Integrate: $\int \sqrt{x^3}\, dx$

A. $\frac{2}{5}x^{5/2} + C$ B. $\frac{5}{2}x^{5/2} + C$

C. $2x^{1/2} + C$ D. $\frac{3}{4}x^{4/3} + C$

$x^{3/2}$

50. Use u substitution to find the indefinite integral for $\int x^2(x^3 + 5)^6\, dx$.

$u = x^3 + 5$

$du = 3x^2\, dx$

$\frac{1}{3}\int u^6$

A. $\frac{1}{21}(x^3 + 5)^7 + C$

B. $\frac{1}{7}(x^3 + 5)^7 + C$

C. $\frac{x^3}{3}\left(\frac{x^4}{4} + 5x\right)^6 + C$

D. $\frac{1}{3}(x^3 + 5)^7 + C$