Review Name Key

1. Find the absolute maximum and absolute minimum values of the function $f(x) = x^3 - 3x^2 + 3$ on the interval $[-2, 3]$, and the points where these values are achieved (if they exist, if not write DNE)

$f'(x) = 3x^2 - 6x$

$0 = 3x(x-2)$

$f(-2) = -17$

$f(0) = 3$

$f(2) = -1$

$f(3) = 3$

minimum $(-2, -17)$

maximum $(0, 3)$ $(3, 3)$

2. Suppose that $f(x)$ and $g(x)$ are differentiable functions, with the following values and derivatives:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	1/2	0	5	1
2	1/3	3	1/4	-2
3	-2	2	-3	-5
4	-1	1	7	3

And let $h(x) = f(g(x^2))$. Find $h'(2)$.

$h'(x) = f'(g(x^2)) \cdot g'(x^2) \cdot 2x$

$h'(2) = f'(g(4)) \cdot g'(4) \cdot 4$

$= f'(1) \cdot 12$

$= 60$

3. The curve defined by the equation $(x^2 + y^2)^2 = 4x^2 y$ is called a bifolium. Verify that the point $(1, 1)$ lies on the curve, and then compute the slope of the tangent line to the curve at this point. $m = 0$

$(1+1)^2 = 4(1)(1)$ $4 = 4$ ✓

$2(x^2+y^2)' \cdot 2x + 2y \frac{dy}{dx} = 4x^2 \frac{dy}{dx} + 8xy$

$(2x^2+2y^2)(2x+2y \frac{dy}{dx}) = 4x^2 \frac{dy}{dx} + 8xy$

$(4x^3 + 4x^2 y \frac{dy}{dx} + 4xy^2 + 4y^3 \frac{dy}{dx}) - 4x^2 \frac{dy}{dx} = 8xy$

$\frac{dy}{dx} = \frac{8xy - 4x^3 - 4xy^2}{4x^2 y + 4y^3 - 4x^2}$

$\frac{dy}{dx} = \frac{0}{4} = 0$

4. Does the Mean Value Theorem apply if $f(x) = \frac{1}{x}$, a = 1, and b = 3? If yes, find c satisfying the theorem, and if no, explain why the theorem doesn't apply.

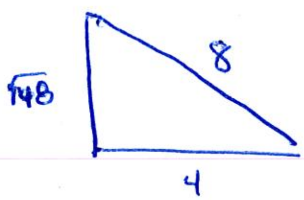
$f'(x) = -x^{-2}$ continuous $[1,3]$

$\frac{f(3)-f(1)}{3-1} = \frac{\frac{1}{3}-1}{2}$ differentiable $[1,3]$

$-\frac{2}{3} \cdot \frac{1}{2} = -\frac{1}{3}$

$\frac{-1}{c^2} = \frac{-1}{3}$ $c = \sqrt{3}$

5. An 8-foot ladder is leaning against a wall, and it begins to slip. The base of the ladder is moving away from the wall at a constant rate of 2 feet per second, and the top of the ladder maintains contact with the wall. What is the speed of the top of the ladder (which is moving down the wall) when the base of the ladder is 4 feet away from the wall?



$\frac{dx}{dt} = 2$ ft/sec

$x^2 + y^2 = 8^2$

$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$

$2(4)(2) + 2\sqrt{48} \cdot \frac{dy}{dt}$

$\frac{dy}{dt} = \frac{-2}{\sqrt{3}}$ ft/sec

6. True or False? The most general antiderivative of $f(x) = \frac{1}{x^2}$ is $F(x) = -\frac{1}{x} + C$.

False $\begin{cases} -\frac{1}{x} + C_1 & x > 0 \\ -\frac{1}{x} + C_2 & x < 0 \end{cases}$

7. True or False? $\int_a^c f(x)\,dx + \int_b^a f(x)\,dx = \int_b^c f(x)\,dx$

True.

8. $\lim_{x \to 0} \frac{x - \sin(x)}{x - \tan(x)} =$

$\frac{x - \sin x}{x - \frac{\sin x}{\cos x}} = \frac{x - \sin x}{\frac{x\cos x - \sin x}{\cos x}} = \frac{(x - \sin x)\cos x}{x \cos x - \sin x}$

9. What is the derivative of $g(x) = \int_0^{x^2} \sin(t)\,dt$?

$2x \sin(x^2)$

10. Sketch the graph of a function $f(x)$ which satisfies the following:

a. $f(0) = 1$

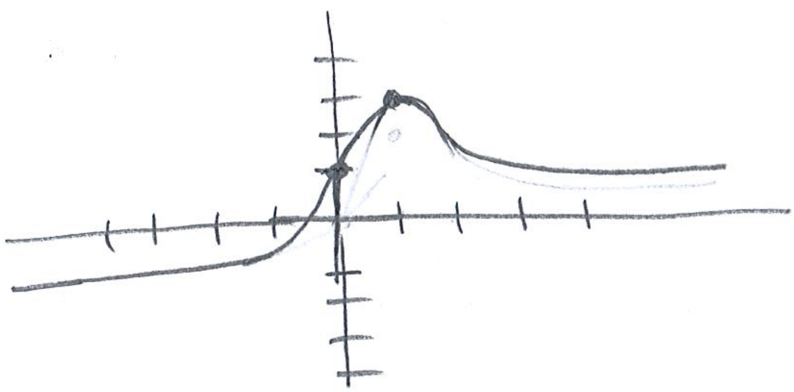
b. $\lim_{x \to -\infty} f(x) = -1$ H. A

c. $\lim_{x \to \infty} f(x) = 1$ H. A

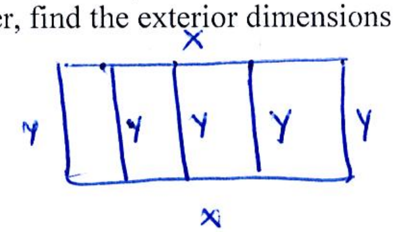
d. $f'(x) > 0$ for x < 1, and $f'(x) < 0$ for x > 1

e. $f''(x) > 0$ for x < 0, and $f''(x) < 0$ for $0 < x < 2$

f. The absolute maximum value of $f(x)$ is 3.



11. Suppose you'd like to build a rectangular enclosure with a total area of 7,000 meters. Moreover, you'll build three walls inside the enclosure which are parallel to one of the sides (splitting it up into four regions) If the price of the exterior walls is $7 per meter, and the price of the three interior walls is $2 per meter, find the exterior dimensions which minimize the cost.



$x \cdot y = 7{,}000$

$y = 7000/x$

$C = 7(2x + 2y) + 2(3y)$

$C = 14x + 20y$

$C = 14x + \frac{140{,}000}{x}$

$C' = 14 - 140{,}000\, x^{-2}$

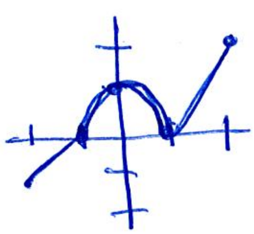
$0 = 14 - \frac{140{,}000}{x^2}$

$14x^2 = 140{,}000$

$x = 100$ m

$y = 70$ m

12. Let $h(x) = \begin{cases} x + 1 & -2 \le x \le -1 \\ \sqrt{1 - x^2} & -1 < x < 1 \\ 2x - 2 & 1 \le x \le 2 \end{cases}$ Sketch its graph: Then use it to compute $\int_{-2}^{2} f(x)\, dx$



$\int_{-2}^{-1} x + 1\, dx + \int_{-1}^{1} \sqrt{1 - x^2}\, dx + \int_{1}^{2} 2x - 2\, dx$

$-\frac{1}{2}(1)(1) + \frac{1}{2}\pi(1) + \frac{1}{2}(1)(2)$

$-\frac{1}{2} + \frac{1}{2}\pi + 1$

$\frac{1}{2}\pi + \frac{1}{2} = \frac{1}{2}(\pi + 1)$

13. Suppose $f''(x) = \sin(x) + \cos(x)$, $f(0) = 3$, and $f'(0) = 4$. Find $f(x)$.

$f'(x) = -\cos x + \sin x + C$

$4 = -1 + 0 + C \quad C = 5$

$f'(x) = -\cos x + \sin x + 5$

$f(x) = -\sin x - \cos x + 5x + C_2 \quad C_2 = 4$

$f(x) = -\sin x - \cos x + 5x + 4$

14. Find the value of c that makes $f(x) = \begin{cases} cx^2+4x & \text{if } x<5 \\ x^3-cx & \text{if } x \geq 5 \end{cases}$ continuous and differentiable.

$\lim\limits_{x \to 5^-} \quad 25c + 20$

$\lim\limits_{x \to 5^+} \quad 125 - 5c$

$25c + 20 = 125 - 5c$

$30c = 105$

$c = 105/30 = 7/2$