Find each of the following and graph the parabola.

$\frac{1}{4a} = \frac{-1}{8}$

1) $y = -2x^2 + 8x - 3$

$y = -2(x^2 - 4x + 4) - 3 + 8$

$y = -2(x-2)^2 + 5$

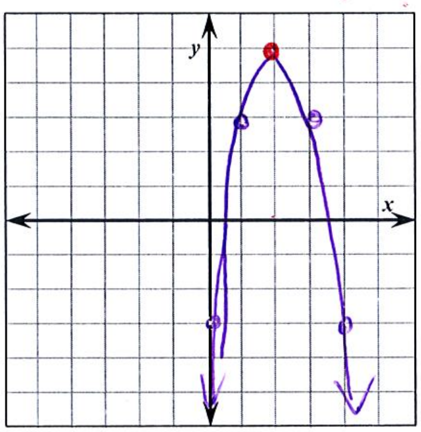
Vertex: $(2, 5)$

Focus: $(2, 4\frac{7}{8})$

Axis of Symmetry: $x = 2$

Directrix: $y = 5\frac{1}{8}$

Direction of Opening: down



2) $x = -\frac{1}{2}y^2 + 2y + 4$

$\frac{1}{4a} = \frac{-1}{2}$

$-\frac{1}{2}(y^2 - 4y + 4) + 4 + 2$

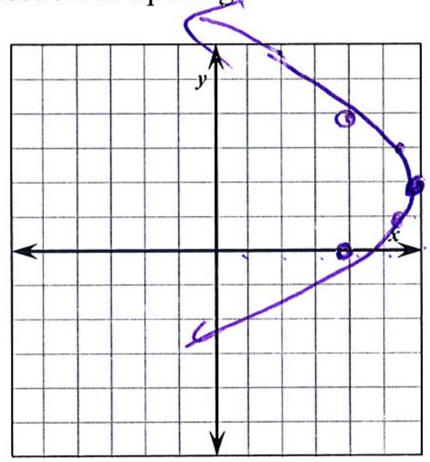
Vertex: $(6, 2)$

Focus: $(5\frac{1}{2}, 2)$

Axis of Symmetry: $y = 2$

Directrix: $x = 6\frac{1}{2}$

Direction of Opening:



Reflective Property of a Parabola: The tangent line to a parabola at point P makes equal angles with the following two lines:

1. The line passing through P and the focus
2. The axis of the parabola

Eccentricty of a Parabola: The eccentricity (how much it deviates from being circular) of a parabola is 1.

Circles- The set of all points in a plane that are equidistant from a given point called the center.

$(x-h)^2 + (y-k)^2 = r^2$

center (h, k) radius = r

Ex: $x^2 + y^2 + 2x - 12y = 35$

$x^2 + 2x + 1 \quad y^2 - 12y + 36 = 35 + 37$

$(x+1)^2 + (y-6)^2 = 72$

Ex: $3x^2 + 3y^2 + 6y + 6x = 2$

$3(x^2 + 2x + 1) + 3(y^2 + 2y + 1) = 2 + 6$

$(x+1)^2 + (y+1)^2 = 8/3$

Ex: Write the equation of the circle whose diameter has end pts (3, 5) and (6, 1).

Diameter = 5 Radius = $5/2$

Center ($\frac{9}{2}$, 3)

$d = \sqrt{(6-3)^2 + (1-5)^2}$

$d = \sqrt{25}$

$d = 5$

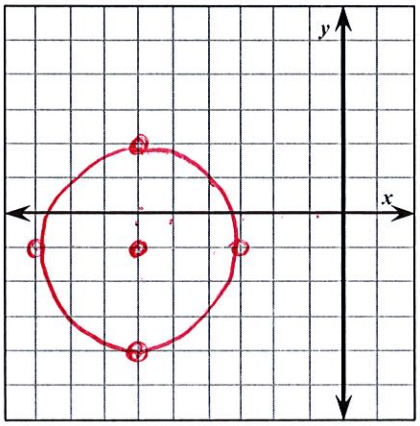
midpt $\left(\frac{3+6}{2}, \frac{5+1}{2}\right)$

$(x - 4.5)^2 + (y-3)^2 = \frac{25}{4}$

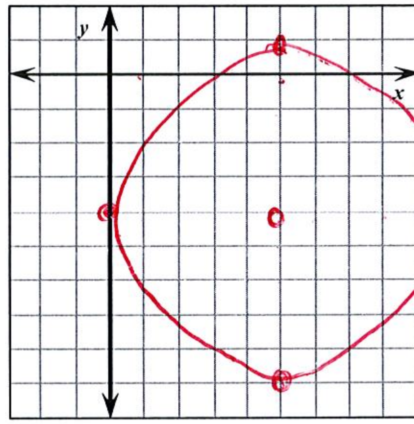
$x^2+12x+36+y^2+2y+1=9$ $x^2-10x+25+y^2+8y+16=25$

Graph each of the following circles.

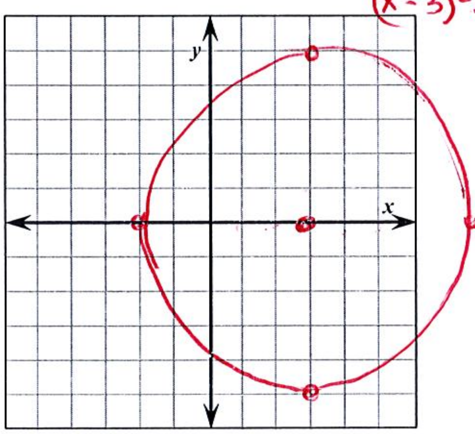
1. $x^2 + 12x + y^2 + 2y = -28$



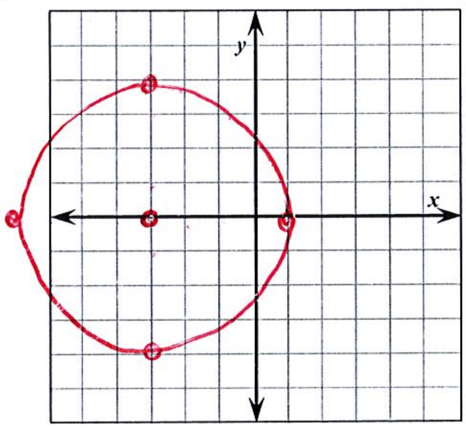
2. $x^2 + y^2 + 8y - 10x + 16 = 0$ $(x-5)^2+(y+4)^2=25$



3. $x^2 + y^2 - 6y - 16 = 0$ $x^2-6y+9+y^2=16$
$(x-3)^2+y^2=25$



4. $(x + 3)^2 + y^2 = 16$



Ellipse: An ellipse is the set of all points in a plane such that the sum of the distances from two given points (foci) is constant.

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \qquad \frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

Horizontal Vertical

Circle $a^2 = b^2 = r^2$

Key $\mathbf{a^2 > b^2}$

If $a^2 = b^2$ then it is a circle

Center (h, k)

Major axis = 2a longer one

Minor axis = 2b

Foci = $\boxed{a^2 - b^2 = c^2}$ c units from center on major axis.

Write the equation in standard form..

Ex: $9x^2 + 25y^2 = 225$

$\frac{x^2}{25} + \frac{y^2}{9} = 1$

Ex: $x^2 + 9y^2 - 4x + 54y + 49 = 0$

$x^2 - 4x \_\_\_ + 9y^2 + 54y = -49$

$x^2 - 4x + 4 + 9(y^2 + 6y + 9) = 36$ (+4, +81)

$\frac{(x-2)^2}{36} + \frac{9(y+3)^2}{36} = \frac{36}{36}$

$\frac{(x-2)^2}{36} + \frac{(y+3)^2}{4} = 1$

Ex: $x^2 + 25y^2 - 8x + 100y + 91 = 0$

$x^2 - 8x + 16 + 25(y^2 + 4y + 4) = -91 + 16 + 100$

$\frac{(x-4)^2}{25} + \frac{(y+1)^2}{1} = 1$

Write the equation:

Ex: The endpoints of major axis (2, 12) & (2, -4)
Endpoints of minor axis are ((4, 4) & (0, 4)

$2a = 16$ $a = 8$ a is vertical

$2b = 4$ $b = 2$ midpt $\left(\frac{2+2}{2}, \frac{12+-4}{2}\right)$

center (2, 6)

$\frac{(x-2)^2}{4} + \frac{(y-4)^2}{64} = 1$

Ex: Foci are at (12, 0) & (-12, 0).
The endpoints of the minor axis are (0, 5) & (0, -5).

center (0, 0) $c = 12$

$b = 5$

$\frac{x^2}{169} + \frac{y^2}{25} = 1$

Ex: $64x^2 + 9y^2 = 576$

$\frac{x^2}{9} + \frac{y^2}{64} = 1$

Ex: $16y^2 + 9x^2 - 96y - 90x + 225 = 0$

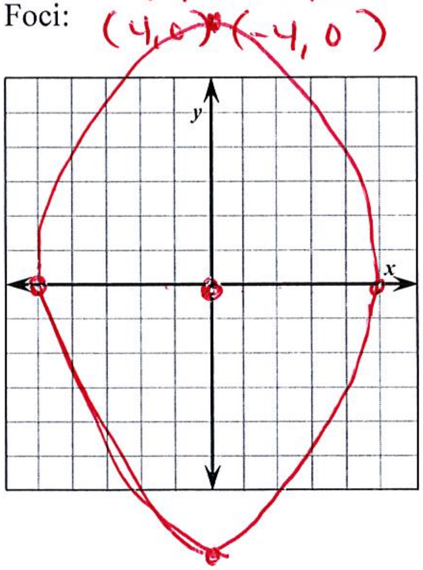
$16(y^2 - 6y + 9) + 9(x^2 - 10x + 25) = 144$

$\frac{(y-3)^2}{9} + \frac{(x-5)^2}{16} = 1$

Graph:

Ex 1: $9x^2 + 25y^2 = 225$

Center: (0, 0)
Vertices: (5, 0) (-5, 0)
Foci: (4, 0) (-4, 0)



Vertices are a units from the center.

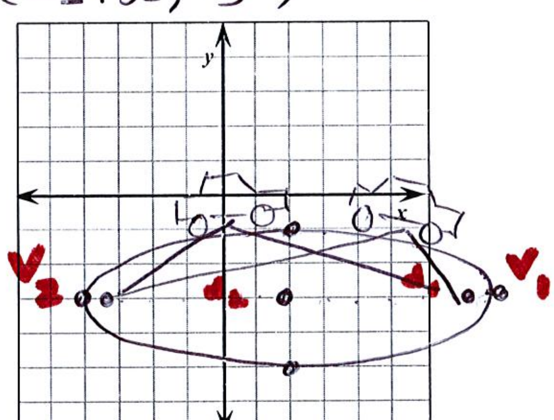
Ex 2: $x^2 + 9y^2 - 4x + 54y + 49 = 0$

$\frac{(x-2)^2}{36} + \frac{(y+3)^2}{4} = 1$

$a = 6$ $b = 2$

major 12 minor 4

Center: (2, -3)
Vertices: (-4, -3) (8, -3)
Foci: $(2 \pm \sqrt{32}, -3)$



$a^2 - b^2 = c^2$

$32 = c^2$ $c = \pm\sqrt{32}$

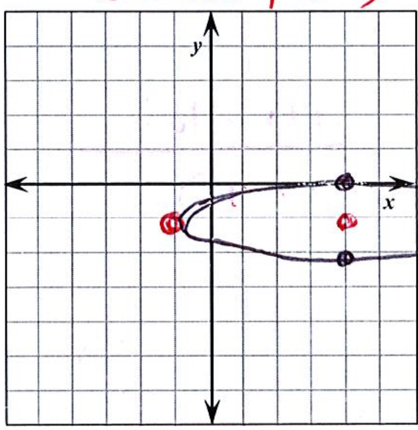
Ex 3: $x^2 + 25y^2 - 8x + 100y + 91 = 0$

$x^2 - 8x + 16 + 25(y^2 + 4y + 4) =$

Center: $(4, -1)$

Vertices: $(9, -1)\ (-1, -1)$

Foci: $(4 \pm \sqrt{24}, -1)$



Ex 4: $16y^2 + 9x^2 - 96y - 90x + 225 = 0$

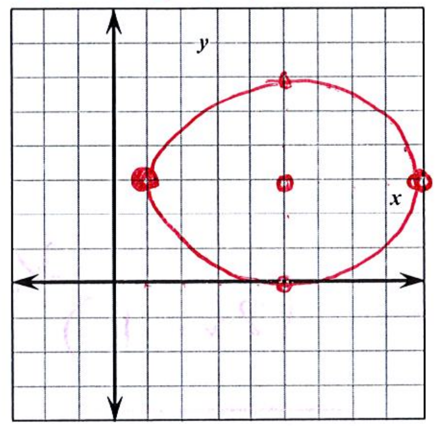
$\frac{(y-3)^2}{9} + \frac{(x-5)^2}{16} = 1$

Center: $(5, 3)$

Vertices: $(9, 3)\ (1, 3)$

Foci: $(5 \pm \sqrt{7}, 3)$

$e = \frac{c}{a} = \frac{\sqrt{7}}{4}$



Hyperbola- Set of all points in a plane such that the absolute value of the difference of the distance from any point on the Hyperbola to two given points (foci) is constant.

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \qquad \frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

Center (h, k)

key a^2 comes first.

Vertices- a units from center

Asymptotes- as a hyperbola recedes from the center the branches approach lines called asymptotes.

Transverse axis = 2a

Conjugate axis= 2b

$\boxed{\text{Foci} \quad a^2 + b^2 = c^2}$

Ex: $\frac{x^2}{25} - \frac{y^2}{49} = 1$

$a = 5$

$b = 7$

center $(0, 0)$

Ex: $25x^2 - 4y^2 + 100x + 24y - 36 = 0$

$25(x^2 + 4x + 4) - 4(y^2 - 6y + 9) = 25$

$\frac{(x+2)^2}{1} - \frac{(y-3)^2}{\frac{25}{4}} = 1$

Ex: $y^2 - 4x^2 + 6y + 8x = 59$

$(y-3)^2 - 4(x-1)^2 = 60$

$\frac{(y-3)^2}{60} - \frac{(x-1)^2}{15} = 1$

Ex: $16x^2 - y^2 + 96x + 8y + 112 = 0$

$$144(y^2-4y+4)-25(x^2+6x+9)=\begin{array}{r}3249\\576\\-225\end{array}$$

Ex: $144y^2 - 25x^2 - 576y - 150x = 3{,}249$

$$\frac{(y-2)^2}{25}-\frac{(x+3)^2}{144}=1$$

Ex1: $\frac{x^2}{4}-\frac{y^2}{9}=1$ center $(0,0)$

$a^2=4$ $b^2=9$

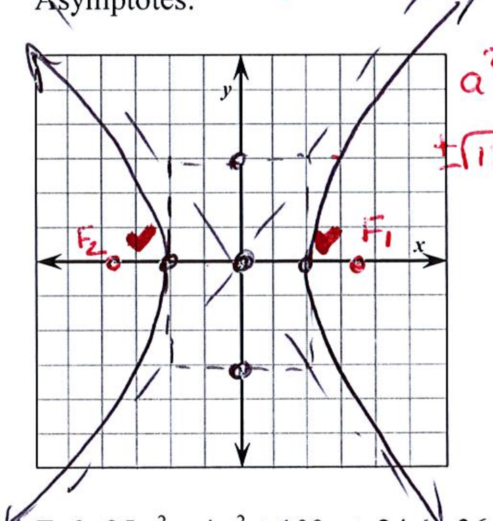
Vertical or Horizontal: $a=2$ $b=3$

Center: Vertices:

Foci: $(0\pm\sqrt{13},0)$

$-3/2$ $3/2$

Asymptotes:



$a^2+b^2=c^2$

$\pm\sqrt{13}=c$

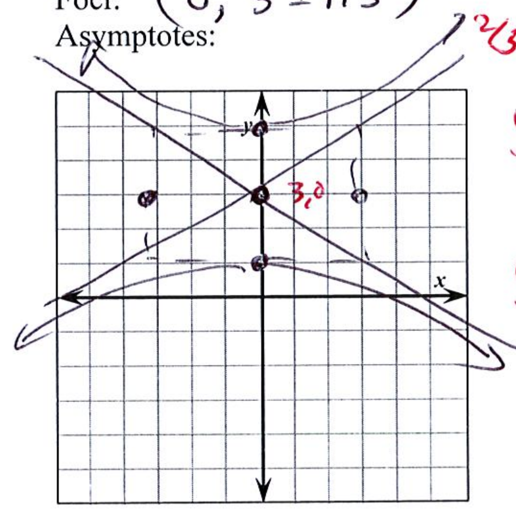
Ex 2: $\frac{(y-3)^2}{4}-\frac{x^2}{9}=1$

Vertical or Horizontal: center $(0,3)$

Center: Vertices:

Foci: $(0,3\pm\sqrt{13})$

Asymptotes: $2/3$



$y=2/3x+b$

$y=-2/3x+b$

Ex3: $25x^2 - 4y^2 + 100x + 24y - 36 = 0$

$25(x^2+4x+4)-4(y^2-6y+9)=100$

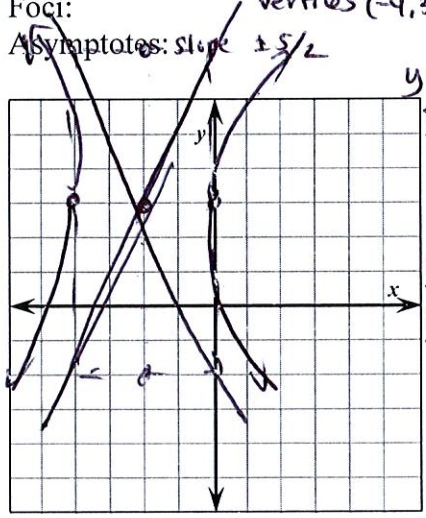
$\frac{(x+2)^2}{4}-\frac{(y-3)^2}{25}=1$

Vertical or Horizontal:

Center: Vertices: center $(-2,3)$

Foci: vertices $(-4,3)(0,3)$

Asymptotes: slope $\pm 5/2$



$y=5/2x+b$

$3=5/2(-2)+b$

$b=8$

$y=5/2x+8$

$y=-5/2x+b$

$y=-5/2(-2)+b$

$b=-2$

$y=-5/2x-2$

$a^2+b^2=c^2$

$c=\pm\sqrt{29}$

Foci $(-2\pm\sqrt{29},3)$

Ex 4: $y^2 - 4x^2 + 6y + 8x = 59$

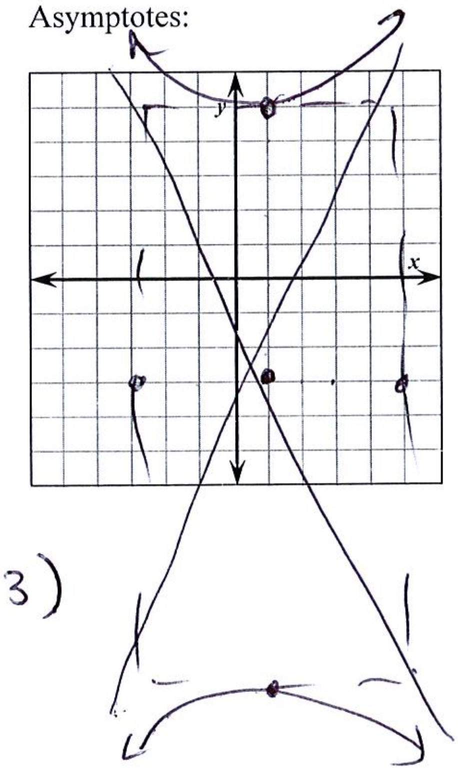
Vertical or Horizontal:

Center: Vertices: center $(1,-3)$

$\frac{(y+3)^2}{64}-\frac{(x-1)^2}{16}=1$

Foci:

Asymptotes:



Write the equation of a hyperbola with the following characteristics:

5: The asymptotes: $y = \pm\frac{5}{12}x$; focus (13, 0) center (0,0)

$\frac{rise}{run}$ a = 12

c is horizontal thus a is two

a = 12
b = 5

$\frac{x^2}{144} - \frac{y^2}{25} = 1$

6: Center (2, -3); Vertex (5, -3); Focus (-10, -3)

c v

a = 3

horizontal

Other vertex (-1, -3)

thus c = 12

$9 + b^2 = 81$

$\frac{(x-2)^2}{9} - \frac{(y+3)^2}{135} = 1$

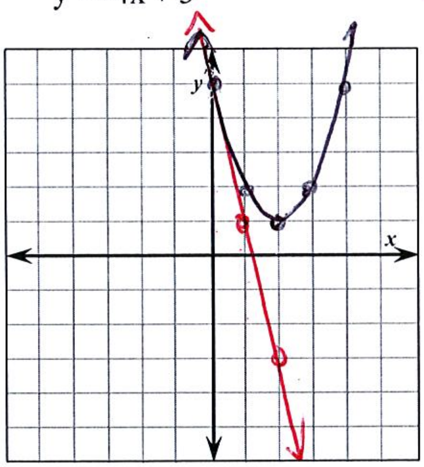
7: Center (-6, -1); a = 4; b = 1; major axis is horizontal

$\frac{(x+6)^2}{16} - \frac{(y+1)^2}{1} = 1$

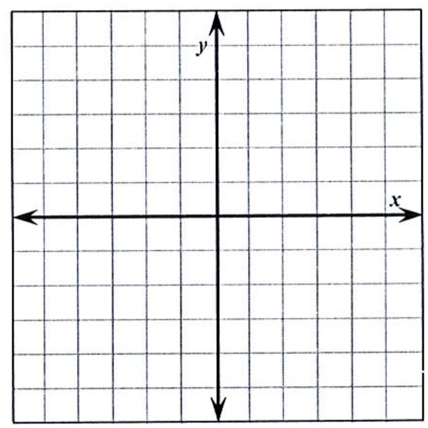
Graphing Quadratic Functions:

Ex: $y = (x-2)^2 + 1$
$y = -4x + 5$

Sol: (0, 5)



Ex: $4x^2 - y^2 = 36$
$(x-5)^2 + y^2 = 64$



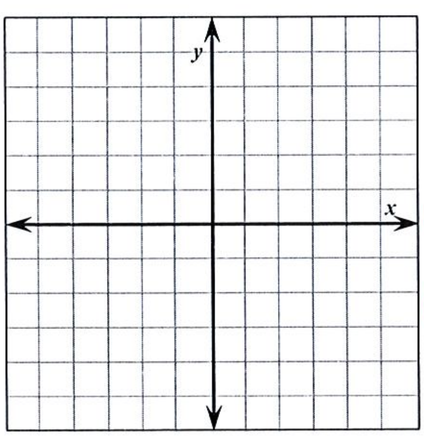
Ex: $5x^2 + y^2 = 30$ (2)
$6x^2 - 2y^2 = 4$

$10x^2 + 2y^2 = 60$
$6x^2 - 2y^2 = 4$
$16x^2 = 64$
$x = \pm 2$

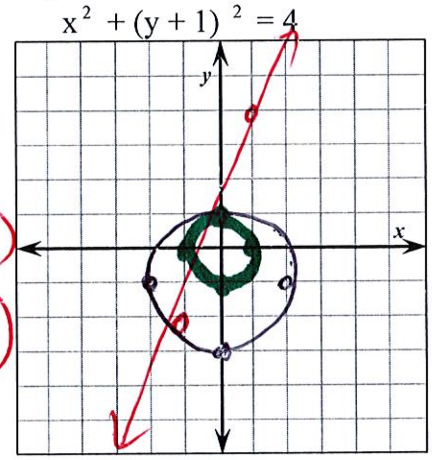
$20 + y^2 = 30$
$y = \pm\sqrt{10}$

$(2, \sqrt{10})$ $(2, -\sqrt{10})$

$(-2, \sqrt{10})$ $(-2, -\sqrt{10})$



Ex: $x^2 + y^2 = 1$ unit circle
$y = 3x + 1$
$x^2 + (y+1)^2 = 4$



Solution (0, 1)