The Chain Rule: If $h(x) = f(g(x))$ then $h'(x) = f'(g(x)) \cdot g'(x)$

Find the derivative:

1. $y = \frac{1}{4\sin(2x-3)}$

$\frac{-\cos(2x-3)}{2\sin^2(2x-3)}$

2. $f(\theta) = \theta + 2\tan\sqrt[3]{\theta}$

$1 + 2\sec^2\sqrt[3]{\theta} \cdot 1/3\,\theta^{-2/3}$

$1 + \frac{2\sec^2\sqrt[3]{\theta}}{3\sqrt[3]{\theta^2}} = \frac{3\sqrt[3]{\theta^2} + 2\sec^2\sqrt[3]{\theta}}{3\sqrt[3]{\theta^2}}$

3. $g(z) = \sqrt[3]{2z-1}$

$\frac{2}{3\sqrt[3]{(2z-1)^2}}$

4. $h(\alpha) = (4\alpha\cos\alpha)^3$

$3(4\alpha\cos\alpha)^2 \cdot [-4\alpha\sin\alpha + 4\cos\alpha]$

$-192\alpha^3\cos\alpha\sin\alpha + 192\alpha\cos^3\alpha$

5. $f(x) = (4x+1)^2(x-7)^3$

$2(4x+1)\cdot 4(x-7)^3 + 3(x-7)^2 \cdot (4x+1)^2$

$8(4x+1)(x-7)^3 + 3(x-7)^2(4x+1)^2$

6. $g(x) = \frac{(x-3)^2}{\sqrt{x+1}}$

$\frac{\sqrt{x+1}\cdot 2(x-3) - (x-3)^2\, 1/2(x+1)^{-1/2}}{x+1}$

$= \frac{3x^2-2x-21}{2\sqrt{x+1}\,(x+1)}$

7. $f(x) = \left(\frac{2x-5}{3-x}\right)^3$

$3\left(\frac{2x-5}{3-x}\right)^2 \cdot \frac{2(3-x)+(2x-5)}{(3-x)^2} = \frac{3(2x-5)^2}{(3-x)^4}$

8. $f(x) = \sin(2x+4)^3$

$\cos(2x+4)^3 \cdot 3(2x+4)^2 \cdot 2$

$6(2x+4)^2\cos(2x+4)^3$

$(24x^2+96x+96)\cos(2x+4)^3$

9. $f(x) = x^5(\sec x^2)^2$

$x^5 \cdot 2\sec(x^2)\cdot\sec(x^2)\tan(x^2)\cdot 2x + 5x^4\sec^2(x^2)$

$4x^6\sec^2(x^2)\tan(x^2) + 5x^4\sec^2(x^2)$

10. $f(x) = (\tan x)^3 + \tan x^2$

$3(\tan x)^2 \cdot \sec^2 x + \sec(x^2)\cdot 2x$

$3\tan^2 x\sec^2 x + 2x\sec(x^2)$

11. $f(x) = \sqrt[3]{\sin x + \cos x}$

$1/3(\sin x + \cos x)^{-2/3} \cdot [\cos x - \sin x]$

$= \frac{\cos x - \sin x}{3\sqrt[3]{\sin x + \cos x}}$

12. $f(x) = \left(\frac{1}{x+\pi}\right)^2$ $(x+\pi)^{-2}$

$-2(x+\pi)^{-3}\cdot 1 = \frac{-2}{(x+\pi)^3}$

13. $f(x) = \left(\csc\left(\frac{x}{5}\right)\right)^3$

$3\csc^2(x/5) \cdot -\csc\left(\frac{x}{5}\right)\cot\left(\frac{x}{5}\right)\cdot\frac{1}{5}$

$= -3/5\csc^3(x/5)\cot(x/5)$

14. $f(x) = \pi^2(\sec(\pi x - 1))^2$

$2\pi^2(\sec(\pi x-1))\cdot\sec(\pi x-1)\tan(\pi x-1)\cdot\pi$

$2\pi^3\sec^2(\pi x-1)\tan(\pi x-1)$

15. $y = x^2\cot\frac{1}{x}$

$x^2 \cdot -\csc^2(1/x) \cdot -x^{-2} + 2x\cot(1/x)$

$\csc^2\frac{1}{x} + 2x\cot(1/x)$

16. $f(x) = \csc(2x)\cot(2x)$

$-\csc(2x)\csc^2(2x)\cdot 2 - \cot(2x)\csc(2x)\cdot\cot(2x)\cdot 2$

$-2\csc^3(2x) - 2\csc(2x)\cot^2(2x)$

$-2\csc(2x)[\csc^2(2x) - \cot^2(2x)] = -2\csc(2x)$

Find $f''(x)$ for # 17 - 21

17. $f(x) = 2(x^2-1)^3$

$6(x^2-1)^2 \cdot 2x$

$12x(x^2-1)^2 = 12(x^2-1)\cdot(5x^2-1)$

18. $f(x) = \sin(x^2)$

$f' = 2x\cos(x^2)$

$f'' = 4x^2\sin(x^2) + 2\cos(x^2)$

19. $f(x) = \tan(2x)$ at $\left(\frac{\pi}{6}, \sqrt{3}\right)$

$y' = 2\sec^2(2x)$

$y'' = 2\cdot 4\sec(2x)\sec(2x)\tan(2x)$

$= 8\sec^2(2x)\tan(2x)$ at $\pi/6$

$24\sqrt{3}$

20. $f(x) = (\sin x)^2$

$-2\sin^2 x + 2\cos^2 x$

or $2\cos(2x)$

21. $h(x) = f(g(x))$

$h'(x) = f'(g(x)) \cdot g'(x)$

$h''(x) = f'(g(x))\cdot g''(x) + g'(x)\cdot f''(g(x))\,g'(x)$

$= f'(g(x))\cdot g''(x) + f''(g(x))\cdot[g'(x)]^2$

22. Find the equation of the tangent line to the curve at the indicated point.

a) $s(t) = \sqrt{t^2 + 2t + 8}$ at x=2 $(2,4)$

$\frac{1}{2}(t^2+2t+8)^{-1/2} \cdot 2t+2$

$s' = \frac{t+1}{\sqrt{t^2+2t+8}}$ $s'(2) = \frac{3}{\sqrt{4+4+8}} = \frac{3}{4}$

$y-4 = \frac{3}{4}(x-2)$

b) $f(t) = \frac{3t+2}{t-1}$ at $(0,-2)$

$f' = \frac{(t-1)\cdot 3 - (3t+2)}{(t-1)^2}$

$f'(0) = -5$

$y+2 = -5x$

23. Determine the points in $(0,2\pi)$ at which the graph of $f(x) = 2\cos x + \sin(2x)$ has a horizontal tangent line.

$f' = -2\sin x + 2\cos(2x)$

$0 = -2\sin x + 2[\cos^2 x - \sin^2 x]$

$-2\sin x + 2[1 - 2\sin^2 x]$

$4\sin^2 x + 2\sin x - 2 = 0$

$2(2\sin x - 1)(\sin x + 1) = 0$

$\pi/6 \quad 5\pi/6 \quad 3\pi/2$

24. Find the equation of the normal line to the curve $y = 2\tan\left(\frac{\pi x}{4}\right)$ at $x = 1$. $(1, \sqrt{2})$

$y' = \pi/4 \cdot 2\sec^2(\pi/4\, x)$

$y' = \pi/2 \sec^2(\pi/4) = \pi$

$y - \sqrt{2} = \frac{-1}{\pi}(x-1)$

25. If $g(5) = -3$, $g'(5) = 6$, $h(5) = 3, and h'(5) = -2$ $find\ f'(5)$ if possible. If not tell what the missing information is.

$f' = \frac{hg' - gh'}{h^2}$ $f' = gh' + hg'$

a) $f(x) = \frac{g(x)}{h(x)}$

$\frac{18-6}{9} = \frac{4}{3}$

b) $f(x) = g(h(x))$

$f'(x) = g'(h(x)) \cdot h'(x)$

$= g'(3) \cdot h'(5)$

don't know

c) $f(x) = g(x)h(x)$

$f' = (-3)(-2) + 3(6)$

$6 + 18$

24

d) $f(x) = (g(x))^3$

$f'(x) = 3(g(x))^2 \cdot g'(x)$

$= 3(-3)^2 \cdot 6$

$= 162$

e) $f(x) = g(x + h(x))$

$f' = g'(x+h(x)) \cdot (1 + h'(x))$

$= g'(5 + h(5)) \cdot (1 + h'(5))$

$= g'(8)(1 + -2)$

don't know

f) $f(x) = (g(x) + h(x))^{-2}$

$-2(g+h)^{-3} \cdot g' + h'$

$-2(-3+3)^{-3} \cdot 6 + -2$

$-2(0)^{-3}(4)$

doesn't exist