

Problem Set Optimization AB Fall 2016 5 point each. **Show Work!!**

1. A rectangular plot of land is to be fenced in using two types of fencing. Two opposite sides will use heavy duty fencing selling for \$3 a foot, and the remaining two sides will use standard fencing for \$2 a foot. What are the dimensions of the rectangle of greatest area that can be fenced in at a cost of \$6000?
2. A rectangle has its base on the x axis and its upper two vertices on the parabola $y = 12 - 2x^2$. What is the largest area the rectangle can have?
3. A square sheet of cardboard, 12 inches by 12 inches is used to make an open box by cutting equal squares from each corner and bending up the edges. What size squares should be cut in order to obtain a box with the largest volume?
4. A wire of length 12 inches can be bent into a circle, bent into a square, or cut into two pieces and bent into both. How much wire should be used for the circle if the total area enclosed must be a) a min and b) a max
5. A chemical manufacturer sells sulfuric acid in bulk at a price of \$100 per unit. If the daily total production cost in dollars for x units is $c(x) = 100,000 + 50x + 0.0025x^2$ and if the daily production capacity is at most 7000 units, how many units of sulfuric acid must be manufactured and sold daily to maximize the profit? And would it benefit the manufacturer to expand the daily production capacity?
6. Find the dimensions of the triangle of maximum area that can be inscribed inside a semicircle of radius a, if two vertices lie on the diameter.
7. Show that the right circular cylinder of greatest volume that can be inscribed in a right circular cone has volume that is $\frac{4}{9}$ of the volume of the cone.
8. A commercial cattle ranch currently allows 20 steer per acre of grazing land: on average its steer weigh 2000 lbs. at market. Estimates indicate that the average market weight per steer will be reduced by 50 lbs. for each additional steer added per acre of grazing land. How many steer per acre should be allowed in order for the rancher to get the largest possible total market weight for his cattle?
9. A page of a book is to have an area of 90 square inches with 1 inch margins at the sides and bottom, and $\frac{1}{2}$ inch margins at the top. Find the dimensions that will allow the largest printed area.

10. At 1:00 PM ship A is 30 miles due south of ship B and is sailing north at a rate of 15 miles per hour. If ship B is sailing west at a rate of 10 mph find the time at which the distance between the ships is a minimum.
11. The owner of an apple orchard estimates that if 24 trees are planted per acre, then each mature tree will yield 600 apples per year. For each additional tree planted per acre the number of apples produced by each tree decreases by 12 per year. How many trees should be planted per acre to obtain the most apples per year?
12. A real estate company owns 180 apartments which are fully occupied when the rent is \$300 per month. The company estimates that for each additional \$10 per month increase in rent, 5 apartments will become empty. What rent should be charged in order to obtain the largest gross income?
13. A package can be sent through UPS if the sum of its length and girth is no more than 108. Find the dimensions of the box of maximum volume with a square base that can be sent through UPS.
14. An oil field contains 8 wells, which produce a collective total of 1600 barrels of oil per day. For each additional well that is drilled the average production per well decreases by 10 barrels per day. How many additional wells should be drilled in order to obtain the maximum amount of oil per day from the field?
15. A steel storage tank for propane gas is to be constructed in the shape of a right circular cylinder with a hemisphere on each end. The construction cost per square foot for the end pieces is twice that of the cylindrical piece. If the desired capacity is $10\pi \text{ ft}^3$ what dimensions will minimize the cost?
16. Two corridors 3 ft. and 4 ft. wide respectively meet at a right angle, find the longest non-bendable rod that can be carried horizontally around the corner.
17. A church window consisting of a rectangle topped by a semicircle is to have perimeter P. Find the radius of the semicircle if the area of the window is to be a maximum.
18. A right circular cylinder is generated by rotating a rectangle of perimeter P about one of its sides. What dimensions of the rectangle will generate the cylinder of maximum volume?
19. Jane is 2 miles offshore in a boat and wants to reach cookout which is 6 miles down shore on the beach. She can row 2 mph and walk 5 mph, where should she land the boat in order to reach cookout in the shortest time?
20. The stiffness of a rectangular beam is proportional to its width times the cube of its depth. Find the dimensions of the stiffest beam that can cut from a 12 inch diameter cylindrical log.