Problem Set # 5 Name________________________________

Graphical Applications of the Derivative

1. Given that f is the function defined by $f(x) = \dfrac{x^3 - x}{x^3 - 4x}$.

a. Find the $\lim_{x \to 0} f(x)$.

b. Find the zeros of f.

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c. Write an equation for each vertical and each horizontal asymptote to the graph of f.

Vertical ______________________ Horizontal______________________

d. Describe the symmetry of the graph of f. Show all of your work.

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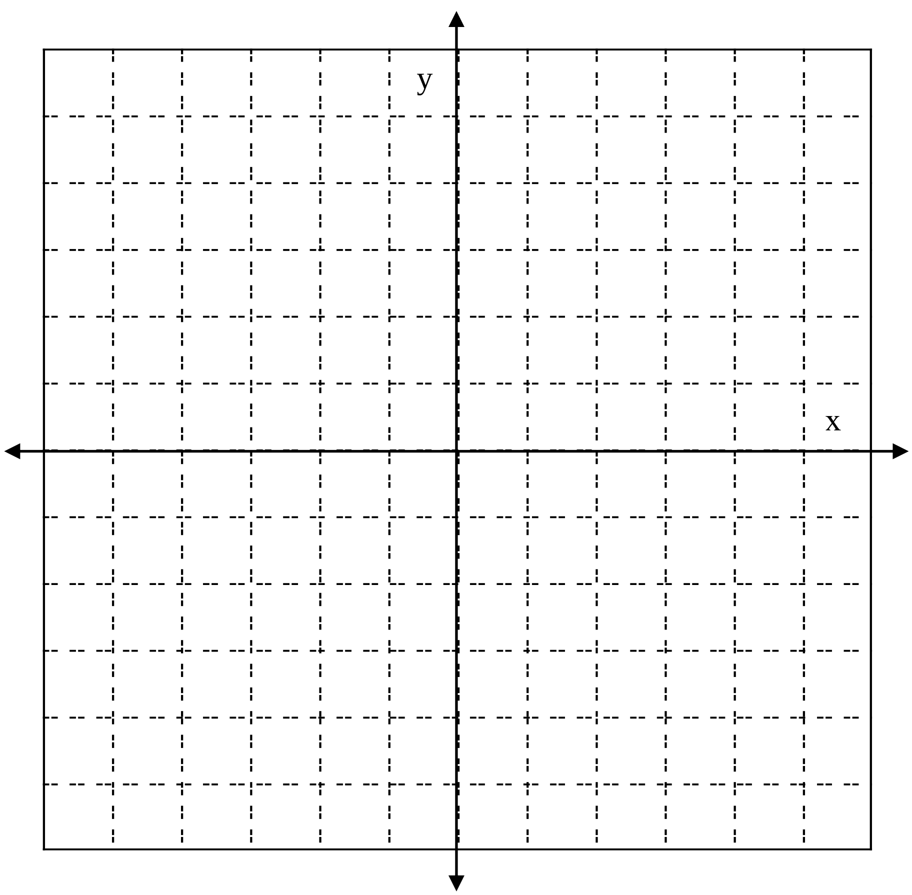
e. Using the information found in parts a, b, c, and d, sketch the graph of f.



2. Let f be the function defined by $f(x) = \sin^2(x) - \sin(x)$ for $0 \leq x \leq \frac{3\pi}{2}$.

a. Find the x-intercepts of the graph of *f*. ____________________

b. Find the intervals on which *f* is increasing. ____________________

c. Find the absolute maximum value and the absolute minimum value of *f*. Justify your answer.

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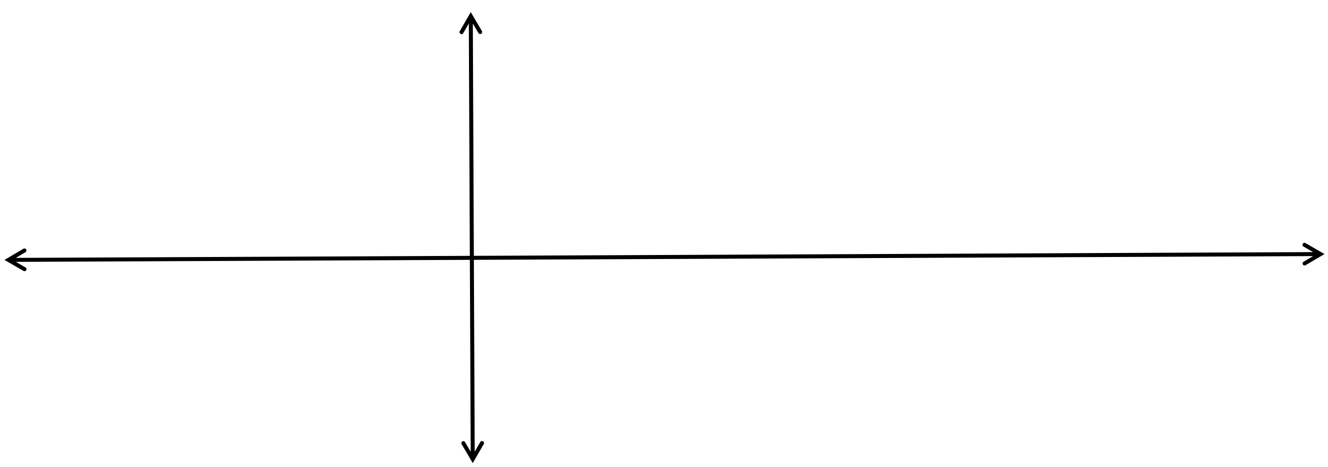
d. Sketch a graph of *f*(x).



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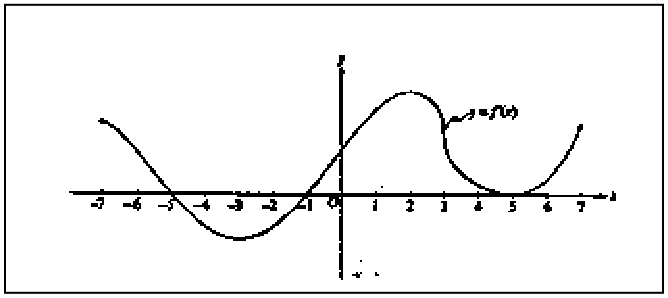

The figure above shows the graph of f', the derivative of the function *f*, for $-7 \le x \le 7$.

The graph of f' has horizontal tangent lines at x = -3, x = 2, and x = 5, and a vertical tangent line at x = 3. Justify all of your answers!

a. Find all values of x, for $-7 \le x \le 7$, at which *f* attains a relative minimum.

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b. Find all values of x, for $-7 \le x \le 7$, at which *f* attains a relative maximum.

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c. Find all values of x, for $-7 \le x \le 7$, at which *f* has a point of inflection.

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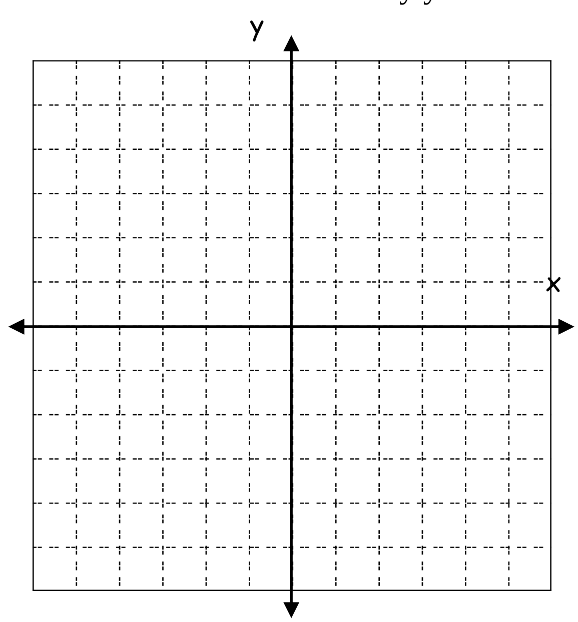
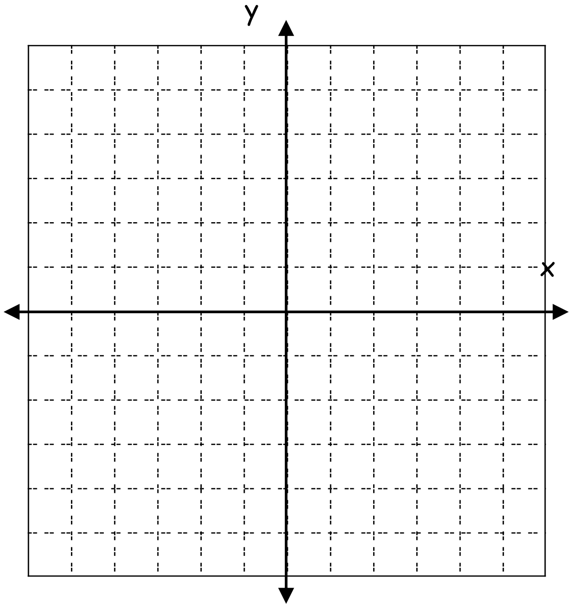
d. At what value of x, for $-7 \le x \le 7$, does *f* attain its absolute maximum? Justify your answer.

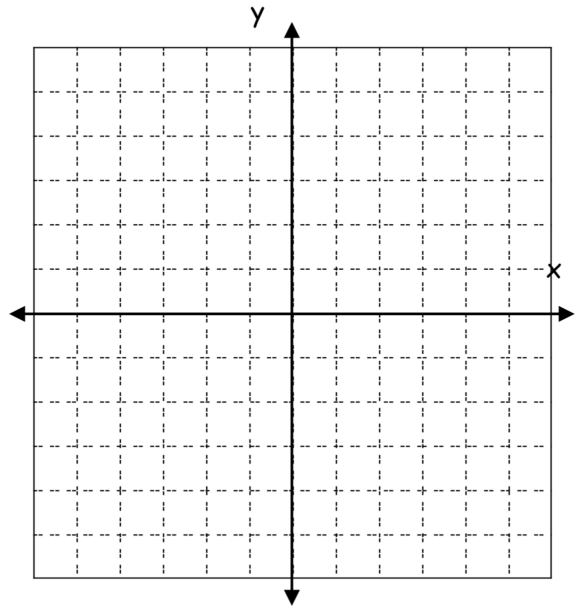
e. If $f(-5) = 0$, make a possible sketch of *f*(x).



4. This problem deals with functions defined by f(x) = x + b sin x, where b is positive and constant and $[-2\pi, 2\pi]$.

a. Sketch the graphs of two of these functions $y = x + \sin(x)$ and $y = x + 3\sin(x)$.





b. Find the x-coordinates of all points, $[-2\pi, 2\pi]$, where the line y = x + b is tangent to the graph of $f(x) = x + b\sin(x)$.

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c. Are the points of tangency described in part (b) relative maximum points of *f*? Why?

d. For all values of b > 0, show that all inflection points of the graph of *f* lie on the line y = x.

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5. Let h be a function defined for all x ≠ 0 such that $h(4) = -3$ and the derivative of *h* is given by $h'(x) = \dfrac{x^2 - 2}{x}$ for all x ≠ 0.

a. Find all values of x for which the graph of *h* has a horizontal tangent, and determine whether *h* has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

b. On what intervals, if any, is the graph of *h* concave up? Justify your answer.

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c. Write an equation for the line tangent to the graph of *h* at x = 4.

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d. Does the line tangent to the graph of *h* at x = 4 lie above or below the graph of *h* for x > 4? Why?

6. Let $f(x) = \begin{cases} 2x - x^2 & \text{if } x \le 1 \\ x^2 + kx + p & \text{if } x > 1 \end{cases}$

a. For what values of *k* and *p* will *f*(x) be differentiable?

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b. For the values of *k* and *p* found in part (a) above, on what interval or intervals is *f*(x) increasing?

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c. Using the values of *k* and *p* found in (a) above, find all points of inflection of the graph of *f*(x). Support your conclusion.