Problem Set # 5 Name\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

Graphical Applications of the Derivative

1. Given that *f* is the function defined by .

 a. Find the. b. Find the zeros of *f*.

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 c. Write an equation for each vertical and each horizontal asymptote to the graph of *f*.

 Vertical \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ Horizontal\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

 d. Describe the symmetry of the graph of *f*. Show all of your work.

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e. Using the information found in parts a, b, c, and d, sketch the graph of *f*.

y

y

y

x

2. Let *f* be the function defined by for .

 a. Find the x-intercepts of the graph of *f*. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

 b. Find the intervals on which *f* is increasing. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

 c. Find the absolute maximum value and the absolute minimum value of *f*. Justify your answer.

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d. Sketch a graph of *f*(x).

3.



 The figure above shows the graph of , the derivative of the function *f*, for –7 < x < 7.

 **The graph of has horizontal tangent lines at x = -3, x = 2, and x = 5,**

 **and a vertical tangent line at x = 3. Justify all of your answers!**

1. Find all values of x, for , at which *f* attains a relative minimum.

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1. Find all values of x, for , at which *f* attains a relative maximum.

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1. Find all values of x, for , at which *f* has a point of inflection.

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1. At what value of x, for , does *f* attain its absolute maximum? Justify your answer.

x

y

1. If , make a possible sketch of *f*(x).

4. This problem deals with functions defined by f(x) = x + b sin x, where b is positive and constant

 and .

1. Sketch the graphs of two of these functions  and .

x

y

x

y

1. Find the x-coordinates of all points, , where the line y = x + b is tangent to the graph of .

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1. Are the points of tangency described in part (b) relative maximum points of *f* ? Why?
2. For all values of b > 0, show that all inflection points of the graph of *f* lie on the line y = x.

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5. Let h be a function defined for all x ≠ 0 such that *h*(4) = -3 and the derivative of *h* is given

 by for all x ≠ 0.

1. Find all values of x for which the graph of *h* has a horizontal tangent, and determine whether *h* has a local maximum, a local minimum, or neither at each of these values. Justify your answers.
2. On what intervals, if any, is the graph of *h* concave up? Justify your answer.

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1. Write an equation for the line tangent to the graph of *h* at x = 4.

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1. Does the line tangent to the graph of *h* at x = 4 lie above or below the graph of *h* for x > 4? Why?
2. Let 
3. For what values of *k* and *p* will *f*(x) be differentiable?

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1. For the values of *k* and *p* found in part (a) above, on what interval or intervals is *f*(x) increasing?

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1. Using the values of *k* and *p* found in (a) above, find all points of inflection of the graph of *f*(x). Support your conclusion.