ICMAB Problem Set 2 Fall 2016

1. A function $f$ is defined on the closed interval from -3 to 3 and has the graph shown below.

a. Sketch the entire graph of $y=|f(x)|$.
b. Sketch the entire graph of $y=f(|x|)$.
c. Sketch the entire graph of $y=f(-x)$.
d. Sketch the entire graph of $y=f\left(\frac{1}{2} x\right)$.
e. Sketch the entire graph of $y=f(x-1)$.
a.

d.

b.

e.

c.

2. A function $f$ is defined for all real numbers and has the following three properties:
(i) $f(1)=5$,
(ii) $f(3)=21$, and
(iii) for all real values of a and $\mathrm{b}, f(a+b)-f(a)=k a b+2 b^{2}$ where k is a fixed real number independent of a and b . Use $\mathrm{a}=1$ and $\mathrm{b}=2$ to find the value of k .
3. Given $f(x)=x^{3}-6 x^{2}+9 x$ and $g(x)=4$.
a. Find the coordinates of the points common to the graphs of f and g .
b. Find all the zeros of $f$.
a. If the domain of $f$ is limited to the closed interval [ 0,2 ], what is the range of $f$ ? Show your reasoning.
4. A manufacturer finds it costs him $x^{2}+5 x+7$ dollars to produce x tons of an item. At production levels above 3 tons, he must hire additional workers, and his costs increase by $3(x-3)$ dollars on his total production. If the price he receives is $\$ 13$ per ton regardless of how much he manufactures and if he has a plant capacity of 10 tons, what level of output maximizes his profits?
5. Given two functions $f$ and $g$ defined by $f(x)=\tan (x)$ and $g(x)=\sqrt{2} \cos x$.
a. Find the coordinates of the point of intersection of the graphs of $f$ and $g$
in the interval $0<x<\frac{\pi}{2}$.
6. Given the function $f$ defined by $f(x)=\ln \left(x^{2}-9\right)$.
a. Describe the symmetry of the graph of $f$.
b. Find the domain of $f$.
c. Find all values of x such that $f(\mathrm{x})=0$.
d. Write a formula for $f^{-1}(x)$, the inverse function of $f$, for $\mathrm{x}>3$.
