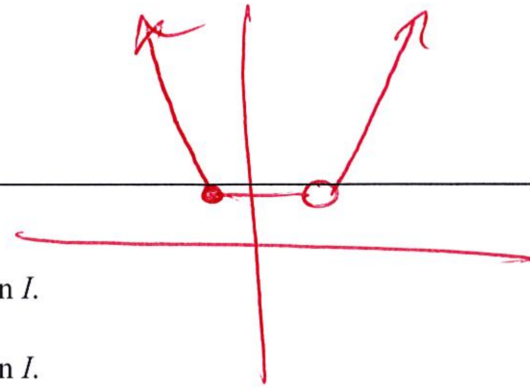
Unit 4 Notes



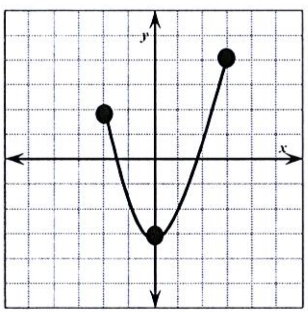
Extrema On An Interval:

Let f be defined on an Interval I containing c.

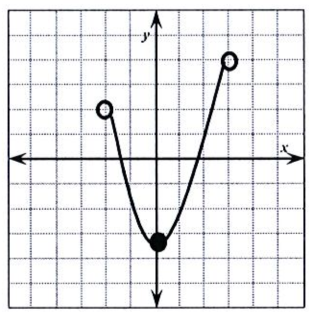
1. f(c) is the **minimum of f on I** if $f(c) \le f(x)$ for all x in I.

2. f(c) is the **m~~in~~**ax**imum of f on I** if $f(c) \ge f(x)$ for all x in I.

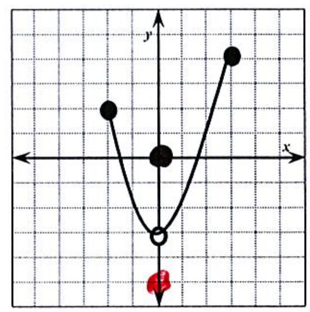
The minimum and maximum of a function on an interval are the extreme values, or extrema, of the function on the interval. The minimum and maximum are also called the **absolute minimum** and **absolute maximum** on the interval.



Max (3, 4)
Min (0, -3)



Max none
Min (0, -3)

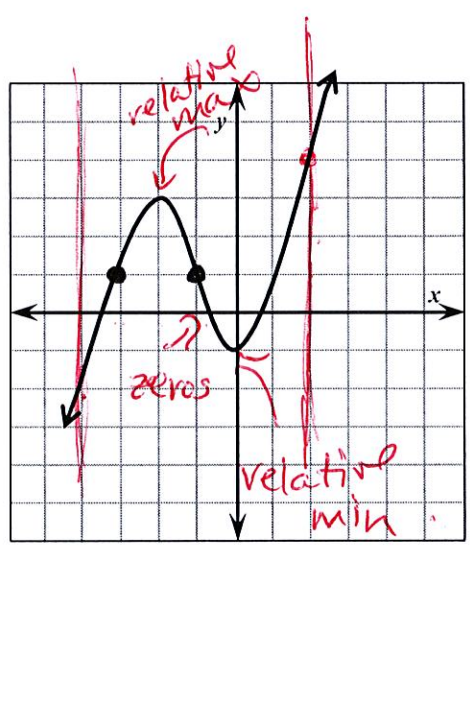


Max (3, 4)
Min none

Extreme Value Theorem: (This is an existence theorem)

If f is continuous on the closed interval $[a, b]$, then f has both a minimum and a maximum on the interval.

Label Parts:



$[-4, 2]$

absolute max $(2, 4)$

absolute min $(-4, -2)$

relative max $(-2, 3)$

relative min $(0, -1)$

Critical Points: Let f be defined at c. If $f'(c) = 0$ or if f is not differentiable at c, then c is a critical number of f.

Relative Extrema occur only at critical numbers. If f has a relative minimum or maximum at $x = c$, then c is a critical number.

Guidelines for Finding Extrema on a Closed Interval

1. Find the critical numbers of f in (a, b)
2. Evaluate f at each critical number in (a, b)
3. Evaluate f at each endpoint of [a, b]
4. The least of these values is the minimum and the greatest is the maximum.

Locate the absolute extrema of the function on the interval.

Ex 1: $y = x^3 + 6x^2 + 9x + 3;\ [-4, 0]$

$y' = 3x^2 + 12x + 9$

$0 = 3(x+3)(x+1)$

Endpt C.P C.P Endpt

$(-4,-1)$ $(-3,3)$ $(-1,-1)$ $(0,3)$

Absolute minima $(-4,-1)(-1,-1)$

Absolute maxima $(0,3)(-3,3)$

Ex 2: $y = \dfrac{x^2}{3x-6};\ [3,6]$ $x \neq 2$

$y' = \dfrac{(3x-6)2x - x^2(3)}{(3x-6)^2} =$ $3x^2 - 12x = 0$

$x = 0$ $x = 4$

Endpt C.P Endpt

$(3, 3)$ $(4, 8/3)$ $(6, 3)$

Absolute minimum $(4, 8/3)$

Absolute maximum $(3,3)$ $(6,3)$

Ex 3: $y = (x)^{\frac{2}{5}};\ [-1, 32]$

$y' = 2/5\, x^{-3/5} = \dfrac{2}{5\sqrt[5]{x^3}}$ $x \neq 0$

Endpt C.P Endpt

$(-1, 1)$ $(0, 0)$ $(32, 4)$

Absolute minimum $(0,0)$

Absolute maximum $(32,4)$

Ex 4: $y = \sin(x) - \cos(x);\ [0, \pi]$

$y' = \cos(x) + \sin(x)$ C.P $= 3\pi/4$

Endpt C.P Endpt

$(0,-1)$ $(3\pi/4, \sqrt{2})$ $(\pi, 1)$

Absolute minimum $(0,-1)$

Absolute maximum $(3\pi/4, \sqrt{2})$

Ex 5: $y = x^3 - 3x^2 - 3;\ (0, 3)$

$y' = 3x^2 - 6x$

$x \cancel{=0}$ $x = 2$

$(2,-7)$

Absolute minimum $(2,-7)$

no maxima

Ex 6: $y = \dfrac{4}{x^2+2};\ (-5, -2]$

$y' = \dfrac{-4(2x)}{(x^2+2)^2}$ $-8x = 0$ $x = 0$

Endpt C.P

$(-2, 2/3)$ $(0, 2)$

no minimum

Absolute max $(-2, 2/3)$

Ex 7: $y = x - \tan(x);\ \left[-\dfrac{\pi}{4}, \dfrac{\pi}{4}\right]$

$y' = 1 - \sec^2(x)$

$1 = \sec^2(x)$ $x = 0$

Endpt C.P Endpt

$(-\pi/4, -\pi/4 + 1)$ $(0, 0)$ $(\pi/4, \pi/4 - 1)$

Abs. min $(\pi/4, \pi/4 - 1)$ Abs. max $(-\pi/4, -\pi/4 + 1)$

Ex 8: $y = 2\sin(x) - \cos(2x);\ [0, 2\pi]$

$y' = 2\cos x + 2\sin(2x)$

$2\cos x + 4\cos(\cancel{2}x)\sin(\cancel{2}x)$

$2(\cos(x))(1 + \sin(x))$

Endpt C.P C.P C.P C.P

$(0,-1)$ $(\pi/2, 3)$ $(7\pi/6, -3/2)$ $(3\pi/2, -1)$ $(11\pi/6, -3/2)$

Endpt $(2\pi, -1)$