Day 5 Notes:

The Chain Rule: : *If* $h(x)=f(g(x))$ then $h'(x)=f'(g(x))\bullet g(x)$

Ex 1: $y=(2-5x^2)^3$

$y' = 3(2-5x^2)^2 \cdot -10x$

$= -30x(2-5x^2)$

or $-750x^5+600x^3-120x$

Ex 2: $y=(2x^3)^4$

$y' = 4(2x^3)^3 \cdot 6x^2$

$= 24x^2(2x^3)^3$

or $192x^{11}$

Ex 3: $y=\sqrt{4x+5}$

$y' = 1/2(4x+5)^{-1/2} \cdot 4$

$= \dfrac{2}{\sqrt{4x+5}}$

Ex 4: $y=\sqrt[3]{2x^2-5x}$

$y' = 1/3(2x^2-5x)^{-2/3} \cdot 4x-5$

$= \dfrac{4x-5}{3\sqrt[3]{(2x^2-5x)^2}}$

Ex 5: $y=(6x+1)^5+\sqrt{2x}$

$y' = 5(6x+1)^4 \cdot 6 + 1/2(2x)^{-1/2} \cdot 2$

$= 30(6x+1)^4 + \dfrac{1}{\sqrt{2x}}$

Ex 6: $y=\sin(x^3)$

$y' = \cos(x^3) \cdot 3x^2$

$= 3x^2\cos(x^3)$

Ex 7: $y=\tan(\sqrt{x})$

$y' = \sec^2(\sqrt{x}) \cdot 1/2\, x^{-1/2}$

$= \dfrac{\sec^2\sqrt{x}}{2\sqrt{x}}$

Ex 8: $y=x^3\cos\left(\dfrac{1}{x}\right)$

$y' = x^3 \cdot -\sin(1/x) \cdot -x^{-2} + 3x^2\cos(1/x)$

$= x\sin(1/x) + 3x^2\cos(1/x)$

Ex 9: $y=\dfrac{(2x+1)^3}{\sqrt{3x-5}}$

$y' = \dfrac{\sqrt{3x-5}\cdot 3(2x+1)^2\cdot 2 - (2x+1)^3 \cdot 1/2(3x-5)^{-1/2}\cdot 3}{3x-5}$

$y' = \dfrac{6\sqrt{3x-5}(2x+1)^2 - 3/2(3x-5)^{-1/2}(2x+1)^3}{3x-5}$

we can simplify.

Ex 10: $y=3x^2\csc(x^4)$

$y' = 3x^2 \cdot -\csc(x^4)\cot(x^4)\cdot 4x^3 + 6x\csc(x^4)$

$= -12x^5\csc(x^4)\cot(x^4) + 6x\csc(x^4)$

Ex 11: $y=\sqrt{4x}\cos(2x)$

$y' = \sqrt{4x}\cdot -\sin(2x)\cdot 2 + 1/2\sqrt{4x}^{-1/2}\cdot 4\cos(2x)$

$= -2\sqrt{4x}\sin(2x) + \dfrac{2\cos(2x)}{\sqrt{4x}}$

Ex 12: $y=3x\sec(5x)$

$y' = 3x\tan(5x)\sec(5x)\cdot 5 + 3\sec(5x)$

$= 15x\tan(5x)\sec(5x) + 3\sec(5x)$

Find the 2nd derivative for each.

Ex 13: $y=(\sin(5x^3))^2$

$y' = 2\sin(5x^3)\cdot\cos(5x^3)\cdot 15x^2$

$= \underbrace{30x^2\sin(5x^3)}_{f}\underbrace{\cos(5x^3)}_{g}$

$y'' = 30x^2\sin(5x^3)\cdot -\sin(5x^3)\cdot 15x^2 + \cos(5x^3)\cdot[450x^4\cos(5x^3)+60x\sin(5x^3)]$

Ex 14: $y=-7(5-3x^2)^5$

$y' = -35(5-3x^2)^4\cdot -6x$

$= \underbrace{210x}_{f}\underbrace{(5-3x^2)^4}_{g}$

$y'' = 210\cdot 4(5-3x^2)^3\cdot -6x + 210(5-3x^2)^4$

$y'' = -5040x(5-3x^2)^3 + 210(5-3x^2)^4$

Ex 15: $y=\sqrt{\sin(2x)+\cos(x^5)}$

$y' = 1/2(\sin(2x)+\cos(x^5))^{-1/2}\cdot[2\cos(2x) - 5x^4\sin(x^5)]$

$= \dfrac{2\cos(2x)-5x^4\sin(x^5)}{2\sqrt{\sin(2x)+\cos(x^5)}}$

y'' = in class.