

14.8 Binomial Probability

1. Conditions of a Binomial Experiment

A binomial experiment exists if and only if these conditions occur:

- The experiment consists of n identical trials.
- Each trial results in one of two possible outcomes.
- The trials are independent.

2. Determine if each situation is binomial or not.

a. A fair coin is tossed 10 times and "heads" or "tails" is recorded each time.

binomial

b. A pair of fair dice is thrown 5 times and the sum of the numbers that come up is recorded each time.

no - more than two outcomes

c. There are 5 red marbles and 6 blue marbles in a bag. One marble is drawn from the bag and its color recorded. The marble is not put back in the bag. A second marble is drawn and its color recorded.

not independent

d. There are 5 red marbles and 6 blue marbles in a bag. One marble is drawn from the bag and its color recorded. The marble is put back in the bag. A second marble is drawn and its color recorded.

binomial

3. Binomial Theorem for Probability Formula:

$$P(k \text{ successes}) = {}_n C_k p^k q^{n-k}$$

where p is the probability of success in one trial and q is the probability of failure

4. While pitching for the Toronto Blue Jays, 4 of every 7 pitches Fireball Roberts threw in the first 5 innings were strikes. What is the probability that in the next inning, Fireball Roberts will throw exactly one strike out of his first five pitches?

$${}_5 C_1 \left(\frac{4}{7}\right)^1 \left(\frac{3}{7}\right)^4$$

5. If there are 10 true/false questions on a quiz, what is the probability that exactly 8 answers are correct?

$${}_{10} C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^2$$

6. If 6 coins are tossed, what is the probability of each?

- a. 3 heads and 3 tails b. at least 4 heads c. 2 heads or 3 tails d. all heads or all tails

$$\begin{aligned}
 & {}_6C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^3 & & {}_6C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^2 + {}_6C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right)^1 + {}_6C_6 \left(\frac{1}{2}\right)^6 & & {}_6C_2 \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^4 + {}_6C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^3 & & {}_6C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^0 + {}_6C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^6
 \end{aligned}$$

7. The probability of Chris making a free throw is 60%. If she shoots five times, what is the probability of each?

- a. all missed b. all made c. exactly 4 made d. at least 3 made

$$\begin{aligned}
 & {}_5C_0 \left(\frac{3}{5}\right)^0 \left(\frac{2}{5}\right)^5 & & {}_5C_5 \left(\frac{3}{5}\right)^5 \left(\frac{2}{5}\right)^0 & & {}_5C_4 \left(\frac{3}{5}\right)^4 \left(\frac{2}{5}\right)^1 & & {}_5C_3 \left(\frac{3}{5}\right)^3 \left(\frac{2}{5}\right)^2 + {}_5C_4 \left(\frac{3}{5}\right)^4 \left(\frac{2}{5}\right)^1 + {}_5C_5 \left(\frac{3}{5}\right)^5 \left(\frac{2}{5}\right)^0
 \end{aligned}$$

8. If you have 6 blue marbles, 4 red marbles, and 5 yellow marbles. Five marbles are being selected. Tell whether each situation is binomial, and if so, find the probability.

- a. P(2 blue); with replacement b. P(2 red); without replacement c. P(1 red, 1 yellow); w/ replacement

$$\begin{aligned}
 & {}_5C_2 \left(\frac{2}{5}\right)^2 \left(\frac{3}{5}\right)^3 & & \text{not} & & \text{not} & & 2 \cdot {}_5C_2 \left[\frac{4}{15} \cdot \frac{5}{15} \cdot \frac{6}{15} \cdot \frac{6}{15} \cdot \frac{6}{15}\right]
 \end{aligned}$$

$10 = \frac{4}{15} \cdot \frac{3}{14} \cdot \frac{11}{13} \cdot \frac{10}{12} \cdot \frac{9}{11}$
 or $\frac{{}_4C_2 \cdot {}_{11}C_3}{{}_{15}C_5}$

9. The recovery rate for a certain cancer patient is 76%. If 12 men were afflicted by the disease, what is the probability that exactly 9 will recover?

$${}_{12}C_9 (.76)^9 (.24)^3 = .2573$$

10. A manufacturing company has a machine that averages one faulty unit for every 1000 it produces. What is the probability that an order of 200 units will have one or more faulty units?

$$1 - P(\text{none faulty}) = 1 - \left[{}_{200}C_0 \left(\frac{999}{1000}\right)^{200} \left(\frac{1}{1000}\right)^0 \right]$$

Using the Calculator to do Binomial Theorem:

Find the probability of 2 successes out of 5 tries if the probability of a success is .3.

Go to 2nd VARS Scroll down to A (binompdf). Enter (5,.3,2). The syntax is binompdf(n, p, x-value)

$${}_5C_2 (.3)^2 (.7)^3$$