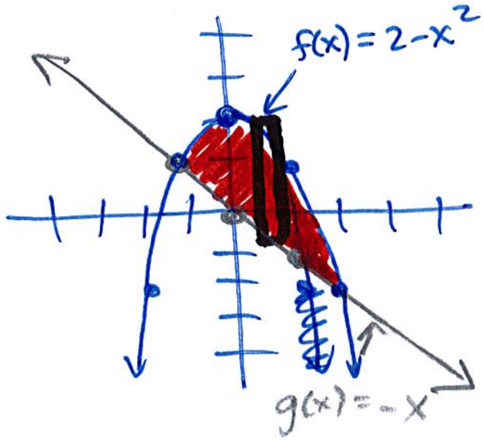
If f and g are continuous function on $[a, b]$ and $g(x) \le f(x)$ for all x in $[a, b]$, then the area of the region bounded by the graphs of f and g and the vertical line x = a and x = b is:

$$\text{Area} = \int_a^b [f(x) - g(x)]\, dx$$

Ex: Find the area of the region bounded by the graphs $y = 2 - x^2$ and the line $y = -x$.
(We must find where they intersect first)

$2 - x^2 = -x \quad 0 = x^2 - x - 2$
$(x-2)(x+1)$



$$\int_{-1}^{2} 2 - x^2 - (-x)\, dx$$

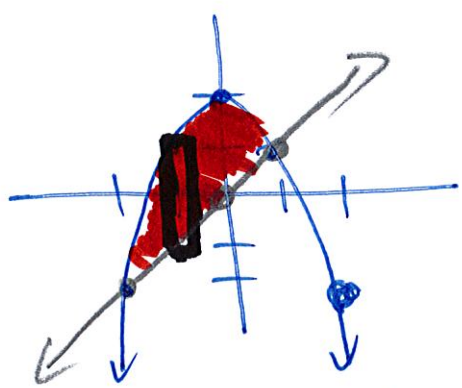
$$\int_{-1}^{2} 2 - x^2 + x\, dx$$

$$2x - \tfrac{1}{3}x^3 + \tfrac{1}{2}x^2 \Big]_{-1}^{2}$$

$$(4 - 8/3 + 2) - [-2 + 1/3 + 1/2]$$

$6 - 8/3$
$18/3 - 8/3$ $\quad 20/6 \quad 10/3 \quad - \quad -2 + 5/6 \quad -7/6 \quad = \frac{27}{6} = \frac{9}{2}$

Ex: Find the area of the region bounded by the graphs $y = 2 - x^2$ and the line $y = x$.



$$\int_{-2}^{1} 2 - x^2 - (+x)\, dx$$

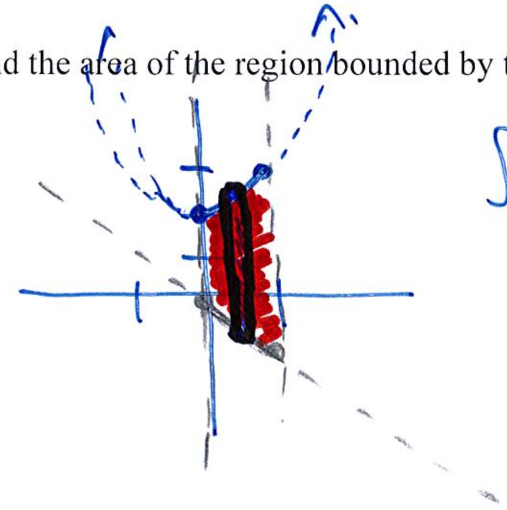
$$\int_{-2}^{1} 2 - x^2 - x\, dx$$

$$2x - \tfrac{1}{3}x^3 - \tfrac{1}{2}x^2 \Big]_{-2}^{1}$$

$$2 - 1/3 - 1/2 - [-4 + 8/3 - 2]$$

$$= 9/2$$

Ex: Find the area of the region bounded by the graphs $y = x^2 + 2$, $y = -x$, x = 0, and x = 1.



$$\int_0^1 x^2 + 2 - (-x)\, dx$$

$$\tfrac{1}{3}x^3 + 2x + \tfrac{1}{2}x^2 \Big]_0^1$$

$$1/3 + 2 + 1/2 = 2\,5/6$$

$$17/6$$

Ex: The sine and cosine curves intersect indefinitely many times, bounding regions of equal areas. Find the area of one of these regions.

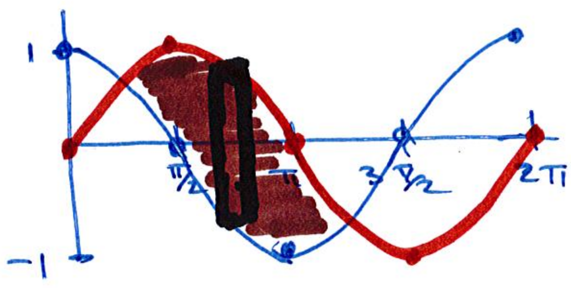
$$\int -\cos x$$

$$\int_{\pi/4}^{5\pi/4} \sin x - \cos x \, dx$$

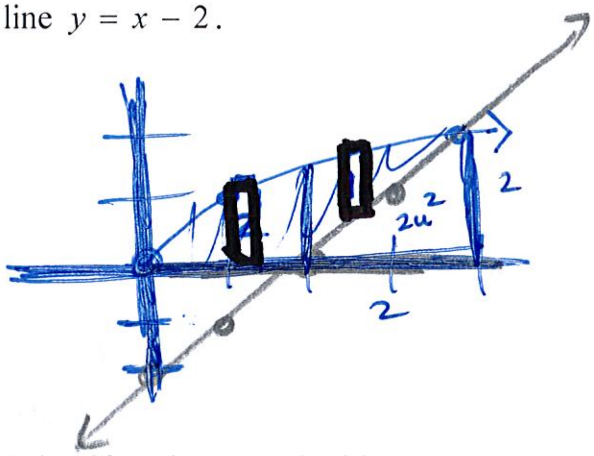
$$-\cos x - \sin x \Big]_{\pi/4}^{5\pi/4}$$

$$\frac{\sqrt{2}}{2} - -\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right)$$

$$\sqrt{2} + \sqrt{2} = 2\sqrt{2}$$



Ex: Find the area in the first quadrant bounded above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$.



$$\int_2^4 \overset{x^{1/2}}{\sqrt{x}} - \overset{-x+2}{(x-2)} \, dx + \int_0^2 \overset{x^{1/2}}{\sqrt{x}} \, dx$$

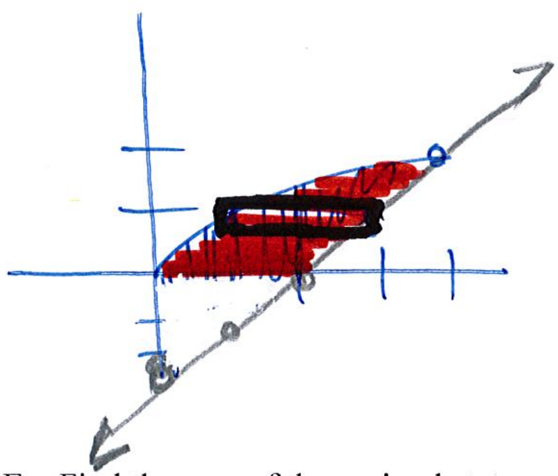
$$\frac{2}{3}x^{3/2} - \frac{1}{2}x^2 + 2x \Big]_2^4 + \frac{2}{3}x^{3/2}\Big]_0^2$$

$$= \frac{10}{3}$$

$$\int_0^4 x^{1/2} \, dx \quad - 2u^2$$

$$\frac{2}{3}x^{3/2} \Big]_0^4$$

Ex: What if we integrated with respect to y?



$y = \sqrt{x}$ $\quad y = x - 2$

$x = y^2$ $\quad x = y + 2$

$$\int_0^2 y + 2 - y^2 \, dy$$

$$\frac{1}{2}y^2 + 2y - \frac{1}{3}y^3 \Big]_0^2$$

$$2 + 4 - \frac{8}{3} = \frac{10}{3}$$

Ex: Find the area of the region between the graphs $y = 3x^3 - x^2 - 10$ and $g(x) = -x^2 + 2x$.

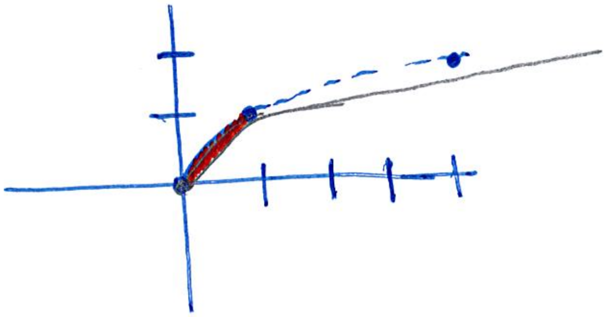
$$3x^3 - x^2 - 10x = -x^2 + 2x$$

$$x = -2, 0, 2$$

$$\int_{-2}^{0} 3x^3 - x^2 - 10x - (-x^2 + 2x) \, dx + \int_0^2 -x^2 + 2x - (3x^3 - x^2 - 10x)$$

$$24u^2$$

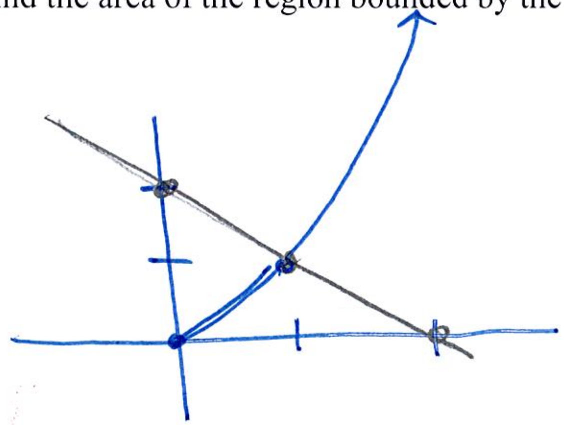
Ex: Find the area of the region bounded by the graphs $x = y^2$, $x = y^3$.



$$\int_0^1 y^2 - y^3 \, dy$$

$$\tfrac{1}{3}y^3 - \tfrac{1}{4}y^4 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}u^2$$

Ex: Find the area of the region bounded by the graphs $y = x^2$, $x + y = 2$.



What makes sense for us to do here?

Ex: Find the area of the region bounded by the graphs $y = 2\cos(x)$, $y = x^2 - 1$.

$$\int_{-1.265424}^{1.265424} 2\cos(x) - (x^2 - 1) \, dx$$

Ex: Find the area of the region bounded by the graphs $y = \cos^2(x)$, $y = 1$.

$$\int_0^{\pi} 1 - \cos^2 x \, dx$$