

Day 3 Warm-Up

A set of 50 quiz scores are normally distributed. The mean of the data is 75 with a standard deviation of 6 points. Using this information, answer the following questions using your calculator:

A. What percent of the scores lie between 63 and 81?



$$81.5\%$$

B. Approximately, how many students scored between 63 and 81?



$$.815 \times 50 \approx 41$$

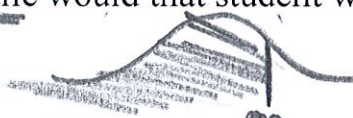
C. How many students would have received a **B** on the quiz?

$$\text{normalcdf}(80, 90, 75, 6) = 19.6\%$$

D. What is the z-score of a student who scored a 90 on the quiz?

$$z_{\text{score}} = \frac{\text{Test} - \text{Mean}}{\text{Standard Dev.}} = \frac{90 - 75}{6} = 2.5$$

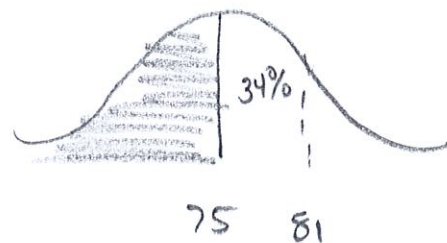
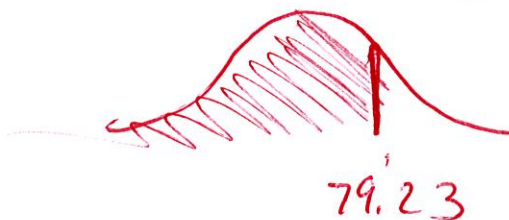
E. What percentile would that student who scored a 90 have been in?



$$\text{normalcdf}(0, 90, 75, 6)$$

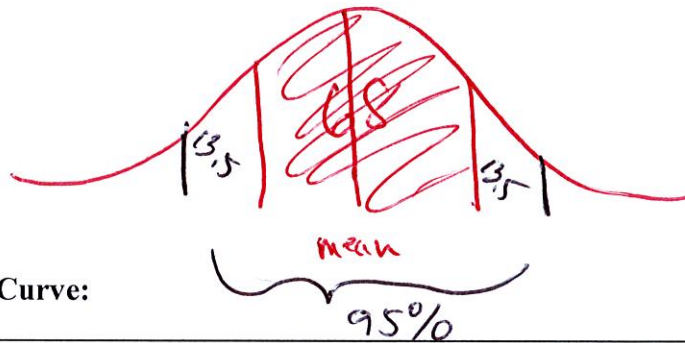
F. A student who scored in the 76<sup>th</sup> percentile would have scored about what on the quiz?

$$\text{invnorm}(.76, 75, 6)$$



15.4 The Normal Distribution

**1. Normal Distribution:** A frequency distribution that often occurs when there is a large number of values in a set. The distribution of data along a bell-shaped, symmetric curve that reaches its maximum height at the mean.



**2. Properties of Normal Curve:**

- 68.3 % of the distribution is within 1 standard deviation of the mean.
- 95.5 of the distribution is within 2 standard deviations of the mean.
- 99.7 % of the distribution is within 3 standard deviations of the mean.

**3. Standard Normal Distribution:** The normal distribution having a mean of 0 and a standard deviation of 1. It is particularly important because any normal distribution can be related to it through the use of standard values. The total area under the curve (and above the x-axis) is 1. Also, the area under the curve to the left of the number  $z$  is the proportion of the data having standard values less than  $z$ .

Ex: Suppose the scores of 500 college freshmen taking Psychology 101 are normally distributed. The mean score is 60 out of 100, and the standard deviation is 10. Sketch a normal curve that represents the frequency scores.



Using the example, estimate how many grades will fall between each of the following intervals:

- a. 50 – 70      68%
- b. 40 – 80      95%
- c. 30 – 90      99%
- d. 55 – 65
- e. 45 – 75      86.6%

normal cdf (low, high, mean, s.d)  
38, 3

Using the above example, estimate each probability:

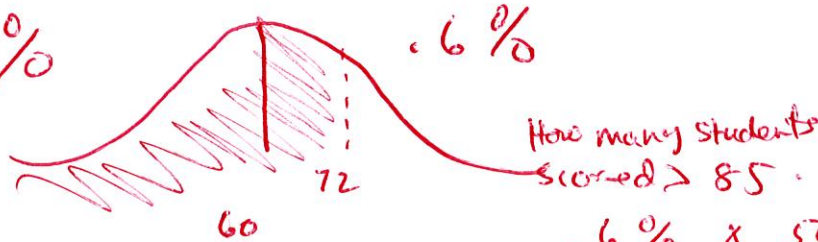
a.  $P(\text{grade} < 72)$

b.  $P(\text{grade} > 85)$

c.  $P(73 < \text{grade} < 83)$

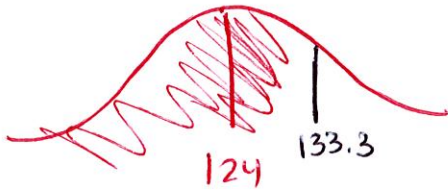
88.5%

normal cdf (85, 100)



$.6\% \times 500 = 3$

Ex: Find the upper and lower limits of the interval about the mean in which 72% of the values of a set of normally distributed data can be found if the mean is 124 and the standard deviation is 16.



invnorm (.14, 124, 16) 106.7

inverse normal cdf (.86, 124, 16) 141.3

Ex: In a normal distribution, about what percent of the data:

a. fall within 2.2 standard deviations of the mean?



$$z = \frac{\text{Test} - \text{Mean}}{\text{Standard Deviation}}$$

normalcdf (-2.2, 2.2, 0, 1) = 97.2

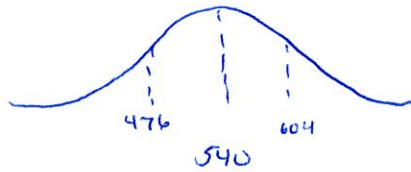
b. do not fall within 2.2 standard deviations of the mean?

2.8%

Ex: A day is selected at random at a post office whose daily letter-handling rate is normally distributed. The mean number of letters per day is 10,000 and the standard deviation is 350. What is the probability that the post office handles between 9,000 and 11,000 letters per day?

99.6%  
normalcdf (9,000, 11,000, 10,000, 350)

1. In a certain large school district, the set of all standardized mathematics scores is normally distributed with mean  $\bar{x} = 540$  and standard deviation of 64. What is the probability that a student chosen at random scores between 580 and 620 on that test?



$$\text{normalcdf}(580, 620, 540, 64)$$

$$16\%$$

2. Suppose 300 values in a set of data were normally distributed.
- a. How many values are within one standard deviation of the mean?

$$300 \times .68 = 204$$

- b. How many values are two standard deviations of the mean?

$$300 \times .95 = 285$$

- c. How many values fall in the interval between the mean and one standard deviation above the mean?

$$300 \times .34 = 102$$

3. **Percentiles:** The value of a variable below which a certain percent of observations fall.

For example, the 20th percentile is the value (or score) below which 20 percent of the observations may be found.

The term percentile and the related term percentile rank are often used in the reporting of scores from norm-referenced tests.

For example, if a score is in the 86th percentile, it is higher than 85% of the other scores.

$$\text{invnorm}(.85)$$