

Solving Exponential and Logarithmic Equations

1. Solve each exponential equation.

a. $6^{x-2} = 6^{3x-4}$

$$x-2 = 3x-4$$

$$2 = 2x$$

$$\mathbf{x = 1}$$

b. $8^{x-1} = \left(\frac{1}{4}\right)^{1-x}$

$$(2^3)^{x-1} = (2^{-2})^{1-x}$$

$$3x-3 = -2+2x$$

$$\mathbf{x = 1}$$

c. $5^{x-2} = \frac{1}{125^x}$

$$5^{x-2} = 5^{-3x}$$

$$x-2 = -3x$$

$$\mathbf{x = 1/2}$$

d. $4^{5-x} = 128$

$$(2^2)^{5-x} = 2^7$$

$$10-2x = 7$$

$$\mathbf{x = 3/2}$$

e. $3^{x-1} = (\sqrt{3})^{x+1}$

$$x-1 = 1/2x + 1/2$$

$$2x-2 = x+1$$

$$\mathbf{x = 3}$$

f. $125^{x-1} = \left(\frac{1}{5}\right)^{1-2x}$

$$3x-3 = -1+2x$$

$$\mathbf{x = 2}$$

g. $2^{3x-1} = 4^{x+2}$

$$3x-1 = 2x+4$$

$$\mathbf{x = 5}$$

h. $7^{2x+4} = \left(\frac{1}{49}\right)^{x-3}$

$$2x+4 = -2x+6$$

$$4x = 2$$

$$\mathbf{x = 1/2}$$

i. $8^{2x-2} = 4^{2-x}$

$$6x-6 = 4-2x$$

$$8x = 10$$

$$\mathbf{x = 5/4}$$

j. $10^x = 3.91$

$$\log 10^x = \log 3.91$$

$$\mathbf{x = \log 3.91}$$

k. $e^x = 5.7$

$$\ln e^x = \ln 5.7$$

$$\mathbf{x = \ln 5.7}$$

l. $5^x = 17$

$$\log 5^x = \log 17$$

$$\mathbf{x = \frac{\log 17}{\log 5}}$$

m. $5e^x = 23$

$$e^x = 23/5$$

$$\ln e^x = \ln 4.6$$

$$\mathbf{x = \ln 4.6}$$

n. $e^{1-5x} = 793$

$$\ln e^{1-5x} = \ln 793$$

$$1-5x = \ln 793$$

$$\mathbf{x = \frac{\ln 793 - 1}{-5}}$$

o. $e^{5x-3} - 2 = 10,476$

$$\ln e^{5x-3} = 10,478$$

$$5x-3 = \ln 10,478$$

$$\mathbf{x = \frac{\ln 10,478}{5} + 3}$$

p. $7^{x+2} = 410$

$$\log 7^{x+2} = \log 410$$

$$(x+2)\log 7 = \log 410$$

$$x\log 7 + 2\log 7 = \log 410$$

$$\mathbf{x = \frac{\log 410 - 2\log 7}{\log 7}}$$

q. $3^{\frac{x}{7}} = .2$

$$\log 3^{\frac{x}{7}} = \log .2$$

$$x\log 3 = 7\log .2$$

$$\mathbf{x = \frac{7\log .2}{\log 3}}$$

r. $e^{4x} - 5e^{2x} - 24 = 0$

$$(e^{2x} - 8)(e^{2x} + 3)$$

$$e^{2x} = 8 \quad e^{2x} = -3$$

$$\mathbf{x = \frac{\ln 8}{2}}$$

~~$$\mathbf{x = \frac{\ln -3}{2}}$$~~

$$s. e^{2x} - 2e^x - 3 = 0$$

$$(e^x + 1)(e^x - 3) = 0$$

$$e^x = -1 \quad e^x = 3$$

$$x = \cancel{\ln -1} \quad x = \ln 3$$

$$t. e^{4x} + 5e^{2x} - 24 = 0$$

$$(e^{2x} + 8)(e^{2x} - 3) = 0$$

$$e^{2x} = -8 \quad e^{2x} = 3$$

$$x = \cancel{\frac{\ln -8}{2}} \quad x = \frac{\ln 3}{2}$$

$$u. e^{4x} - 3e^{2x} - 18 = 0$$

$$(e^{2x} + 3)(e^{2x} - 6) = 0$$

$$e^{2x} = -3 \quad e^{2x} = 6$$

$$x = \cancel{\frac{\ln -3}{2}} \quad x = \frac{\ln 6}{2}$$

$$v. 3^{2x} + 3^x - 2 = 0$$

$$(3^x - 1)(3^x + 2) = 0$$

$$x = \frac{\log 1}{\log 3} = 0$$

$$w. 2^{2x} + 2^x - 12 = 0$$

$$(2^x + 4)(2^x - 3) = 0$$

$$x = \frac{\log 3}{\log 2}$$

2. Solve each logarithmic equation.

a. $\log_3 x = 4$

$$x = 81$$

b. $\log_x(x + 4) = 3$

$$x^3 = x + 4$$

$$x^3 - x - 4 = 0$$

$$x = 1.7963$$

c. $\log_3(x - 4) = -3$

$$x - 4 = \frac{1}{27}$$

$$x = 4.03703$$

d. $\log_5 x + \log_5(4x - 1) = 1$

$$4x^2 - x = 5$$

$$(4x - 5)(x + 1) = 0$$

$$x = 5/4$$

e. $\log_3(x - 5) + \log_3(x + 3) = 2$

$$x^2 - 2x - 24 = 0$$

$$(x - 6)(x + 4) = 0$$

$$x = 6$$

f. $\log_2(x + 2) - \log_2(x - 5) = 3$

$$x + 2 = 8x - 40$$

$$x = 6$$

g. $2\log_3(x + 4) = \log_3 9 + 2$

$$(x + 4)^2 = 81 \quad x + 4 = 9$$

$$x + 4 = 9 \quad \text{and} \quad x + 4 = -9$$

$$x = 5 \quad \text{and} \quad x = \cancel{-13}$$

h. $3\log_2(x - 1) = 5 - \log_2 4$

$$(x - 1)^3 = 8$$

$$x - 1 = 2$$

$$x = 3$$

i. $\log_2(x - 6) + \log_2(x - 4) - \log_2 x = 2$

$$x^2 - 10x + 24 = 4x$$

$$x = 12$$

3. Solve equations involving natural logarithms.

a. $\ln x = 2$

$$e^2 = x$$

b. $5 \ln(2x) = 20$

$$e^4 = 2x \text{ so } x = .5e^4$$

c. $6 + 2 \ln x = 5$

$$e^{-1/2} = x$$

d. $\ln \sqrt{x+3} = 1$

$$e^2 = x + 3$$

$$x = e^2 - 3$$

e. $\ln(x+1) - \ln(x-2) = \ln x^2$

$$x = 2.54$$

4. Solve by graphing each equation.

a. $\log_2 x + \log_2(x+5) = \log_2(x+4)$

$$x^2 + 4x - 4 = 0$$

b. $\ln(x) + \ln(x^2 + 4) = 10$

$$x^3 + 4x - e^{10} = 0$$