Matrices Precalculus

Matrix- System of rows and columns each position in a matrix has a purpose.

Element- Each value in the matrix

a_{23} means the element in the 2nd row, 3rd column

Dimensions- How many rows by number of columns

Ex: $\begin{bmatrix} 5 & 6 & 3 \\ 4 & 2 & 8 \end{bmatrix}$ Ex: $\begin{bmatrix} 3 & 0 & 5 \end{bmatrix}$ Ex: $\begin{bmatrix} -6 & 2 & 4 \\ 0 & 3 & -1 \\ 3 & 4 & 5 \end{bmatrix}$

Identify the a_{12} element:

Name matrices with capital letters.

Row matrix- has one row Column matrix- has one column

Square matrix- same # of rows as columns.

Equal matrices- have the same dimensions and have corresponding elements.

Solve for each variable-

Ex: $\begin{bmatrix} 9 & 4x-1 \\ 12 & -2z \end{bmatrix} = \begin{bmatrix} 3a & 7 \\ -3y & -4 \end{bmatrix}$ Ex: $\begin{bmatrix} 2x+y \\ x-3y \end{bmatrix} = \begin{bmatrix} 6 \\ 31 \end{bmatrix}$

Addition & Subtraction- Matrices must have same dimensions.

Ex: $\begin{bmatrix} 2 & 3 & 6 \\ 4 & 5 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -3 \\ 6 & -1 & 5 \end{bmatrix} =$ Ex: $\begin{bmatrix} 4 & -1 \\ 0 & 5 \\ -3 & 3 \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ 2 & 4 \\ -5 & 3 \end{bmatrix} =$

Ex: Move Quad ABCD 3 units to the left and 5 units up (-1, -4), (3, -5), (2, 2), (6, -1).

Scalar multiplication- multiplying by a constant

Ex: $-2\begin{bmatrix} 3 & 1 & -8 \\ -2 & 4 & 5 \end{bmatrix} =$ Ex: $\frac{1}{2}\begin{bmatrix} 0 & 1 & 3 & 8 \\ -2 & 4 & 6 & 5 \end{bmatrix} =$

Ex: Enlarge $\triangle ABC$ with vertices A(-1, 2), B(-4, -2) and C(3, -1) so that it's perimeter is twice as large as the original figure.

x-coordinate:
y-coordinate:

Day 1 Homework Name ____________________________

A. Perform the indicated operation, if possible. If not possible state the reason.

1. $\begin{bmatrix} 1 & -4 \\ -7 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 5 \\ -5 & 2 \end{bmatrix}$
2. $\begin{bmatrix} -8 & 13 & 24 \\ 1 & -6 & 0 \\ 4 & 5 & -3 \end{bmatrix} - \begin{bmatrix} 20 & -2 & 7 \\ -11 & -5 & 9 \\ 16 & -12 & 5 \end{bmatrix}$
3. $\begin{bmatrix} -2 & 15 \\ 14 & -8 \\ -34 & 5 \end{bmatrix} + \begin{bmatrix} -9 & 12 & 31 \\ -5 & 32 & -27 \end{bmatrix}$

4. $\begin{bmatrix} 1 & 1 \\ \frac{1}{2} & \frac{1}{4} \\ 3 & 8 \end{bmatrix} - \begin{bmatrix} 2 & \frac{3}{4} \\ \frac{1}{2} & 5 \end{bmatrix}$
5. $-4\begin{bmatrix} -1 & 3 & 18 & -7 \end{bmatrix}$
6. $9\begin{bmatrix} -6 & 3 \\ 0 & -12 \\ 10 & -8 \end{bmatrix}$
7. $\frac{1}{2}\begin{bmatrix} -2 & \frac{1}{4} & 7 \\ -\frac{6}{11} & \frac{3}{2} & -\frac{4}{5} \end{bmatrix}$

8. $\begin{bmatrix} 12 & -8 \\ 0 & 5 \\ 0 & 3 \end{bmatrix} - 4\begin{bmatrix} -1 & 0 \\ 3 & -2 \\ -4 & 5 \end{bmatrix}$
9. $3\begin{bmatrix} 7 & -7 \\ -1 & 3 \end{bmatrix} - 5\begin{bmatrix} 2 & -5 \\ 9 & -6 \end{bmatrix}$

Solve for x and y.

10. $\begin{bmatrix} -2x & -8 \\ -10 & -9 \end{bmatrix} = \begin{bmatrix} 6 & y \\ -10 & -9 \end{bmatrix}$
11. $2x\begin{bmatrix} -3 & 4 \\ -11 & 5 \end{bmatrix} = \begin{bmatrix} 12 & -16 \\ y & -20 \end{bmatrix}$

Use the information about 3 Major League Baseball teams' wins and losses in 1998 before and after the All-Star Game.

Before: Atlanta Braves had 59 wins and 29 losses, Seattle Mariners had 37 wins and 51 losses, and Chicago Cubs had 48 wins and 39 losses.

After: Braves had 47 wins and 27 losses, Mariners had 39 wins and 34 losses, and Cubs had 42 wins and 34 losses.

12. Use matrices to organize the information.Write a matrix that shows the total number of wins and losses for the 3 teams in 1998.

Matrix Multiplication – Day 2

1. $\begin{bmatrix} 2 & 4 \\ -3 & 6 \end{bmatrix} \bullet \begin{bmatrix} 1 & -3 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -3 & 21 \end{bmatrix}$

How? 2. $\begin{bmatrix} 4 & 3 \\ 0 & 1 \\ 2 & 4 \end{bmatrix} \bullet \begin{bmatrix} 4 & 5 & 6 & 7 \\ 0 & 1 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 16 & 23 & 30 & 40 \\ 0 & 1 & 2 & 4 \\ 8 & 14 & 20 & 30 \end{bmatrix}$

Let's think about the dimensions first.

Your Turn: Ex: $\begin{bmatrix} 1 & -5 \\ 2 & 4 \end{bmatrix} \bullet \begin{bmatrix} -2 & -3 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} & \\ & \end{bmatrix}$

Ex: $\begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix} \bullet \begin{bmatrix} -3 & 2 \\ -4 & 1 \end{bmatrix} = ?$

Ex: $\begin{bmatrix} 3 & -5 & 7 \end{bmatrix} \bullet \begin{bmatrix} 4 & 2 & -1 \\ 6 & 0 & 3 \\ -3 & -2 & -5 \end{bmatrix} =$

Cummutative Property

Ex: $\begin{bmatrix} -3 & 4 \\ 2 & 5 \end{bmatrix} \bullet \begin{bmatrix} 4 & 5 & -3 \\ 0 & 1 & -2 \end{bmatrix} =$

Ex: What happens when we multiply $\begin{bmatrix} 4 & 5 & -3 \\ 0 & 1 & -2 \end{bmatrix} \bullet \begin{bmatrix} -3 & 4 \\ 2 & 5 \end{bmatrix} =$

Practice:

Ex: $\begin{bmatrix} 1 & -3 \\ -2 & 8 \end{bmatrix} \bullet \begin{bmatrix} -5 & 3 & -2 \\ -1 & 4 & 5 \end{bmatrix} =$

Ex: $\begin{bmatrix} 4 & 2 & -1 \\ 0 & 1 & -5 \end{bmatrix} \bullet \begin{bmatrix} -7 \\ 4 \\ 2 \end{bmatrix} =$

Ex: $\begin{bmatrix} -3 \\ -4 \\ 2 \end{bmatrix} \bullet \begin{bmatrix} 2 & 0 & -6 \end{bmatrix} =$

Ex: $\begin{bmatrix} -4 & -1 \\ 8 & 10 \end{bmatrix} \bullet \begin{bmatrix} -1 & -3 \\ 5 & -4 \end{bmatrix} =$

Unit 9 Matrices Name ________________________________

Day 2 Homework

Determine the dimensions of each matrix M.

1. $A_{8\ x\ 5} \bullet B_{5\ x\ 3} = M$

2. $A_{9\ x\ 4} \bullet M = B_{9\ x\ 1}$

3. $M \bullet A_{1\ x\ 6} = B_{4\ x\ 6}$

Find the product of each, if possible.

4. $\begin{bmatrix} -3 & 4 \\ 5 & 2 \end{bmatrix} \bullet \begin{bmatrix} 1 & 0 \\ 2 & -3 \end{bmatrix}$

5. $\begin{bmatrix} -3 & -1 \\ -2 & 2 \end{bmatrix} \bullet \begin{bmatrix} 5 & 0 \\ 2 & -4 \end{bmatrix}$

6. $\begin{bmatrix} 12 & -6 \end{bmatrix} \bullet \begin{bmatrix} -1 & 0 \\ -4 & 5 \end{bmatrix}$

7. $\begin{bmatrix} -2 \\ -6 \\ 8 \end{bmatrix} \bullet \begin{bmatrix} 2 & 0 & -1 \end{bmatrix}$

8. $\begin{bmatrix} 3 & -2 & 4 \\ 9 & -1 & -5 \end{bmatrix} \bullet \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix}$

9. $\begin{bmatrix} 3 & 5 & -3 \\ 2 & 1 & -2 \end{bmatrix} \bullet \begin{bmatrix} -9 & 0 \\ 1 & 5 \end{bmatrix}$

10. $\begin{bmatrix} -2 & 0 \\ 4 & -1 \\ 3 & 0 \end{bmatrix} \bullet \begin{bmatrix} -5 & -2 \\ -1 & -4 \end{bmatrix}$

11. $\begin{bmatrix} -4 & 1 & -1 \\ 0 & 2 & 3 \\ -3 & -2 & -5 \end{bmatrix} \bullet \begin{bmatrix} -2 & 1 & 0 \\ 3 & -4 & -3 \\ 0 & 2 & 1 \end{bmatrix}$

12. $\begin{bmatrix} 1 & -3 \\ 2 & 0 \end{bmatrix} \bullet \begin{bmatrix} -1 & -2 \\ 5 & -4 \end{bmatrix} + 2\begin{bmatrix} -3 & 5 \\ 0 & 4 \end{bmatrix}$

13. $\begin{bmatrix} -1 & 4 \\ -2 & 0 \end{bmatrix} \bullet \begin{bmatrix} -1 & -2 \\ 5 & -4 \end{bmatrix} - 2X = \begin{bmatrix} 3 & -8 \\ 0 & 4 \end{bmatrix}$

Determinant- square array of #'s or variables enclosed between two parallel vertical bars. Each # or variable is called an element.

2 x 2 Matrix: Rows→ $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ **value is ad – bc**

↑ Columns

Ex- $\begin{vmatrix} 3 & 5 \\ 2 & 6 \end{vmatrix} =$

Ex- $\begin{vmatrix} -2 & 7 \\ 5 & 8 \end{vmatrix} =$

Ex- $\begin{vmatrix} -2 & 8 \\ 6 & 4 \end{vmatrix} =$

Ex- $\begin{vmatrix} \frac{2}{3} & \frac{1}{2} \\ \frac{3}{4} & \frac{-1}{4} \end{vmatrix} =$

3 x 3 Matrix: Move first two columns over and multiply diagonally like 2x2.
Another way is expansion by minors, where the minor of an element is the determinant formed when the row and column containing that element is deleted.

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

Remember: $\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$

Find the determinant of a 3 x 3 matrix.

Ex- $\begin{vmatrix} 3 & 2 & -1 \\ 0 & 4 & 5 \\ 6 & -2 & 3 \end{vmatrix}$

Ex- $\begin{vmatrix} -4 & -2 & -1 \\ 3 & 1 & 2 \\ 0 & 2 & -1 \end{vmatrix}$

Find the determinant of the matrix by using the expansion of minors.

Ex- $\begin{vmatrix} 2 & 3 & -4 \\ 0 & -4 & 5 \\ -1 & -5 & 3 \end{vmatrix}$

Ex- $\begin{bmatrix} 2 & -3 & 1 & 5 \\ 3 & 0 & 1 & 0 \\ 1 & 2 & 1 & -1 \\ 1 & 3 & 3 & -2 \end{bmatrix}$

Inverse of a Square Matrix:

Definition of the Inverse of a Square Matrix
Let A be an $n \times n$ matrix. If there exists a matrix A^{-1} such that $AA^{-1} = I_n = A^{-1}A$

Inverse of a 2 x 2 Matrix:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad \text{then} \qquad A^{-1} = \frac{1}{\det A}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} =$$

$$B = \begin{bmatrix} 4 & -2 \\ 3 & 1 \end{bmatrix} \qquad B^{-1} = \begin{bmatrix} & \\ & \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 4 \\ -1 & 3 \end{bmatrix} \qquad C^{-1} = \begin{bmatrix} & \\ & \end{bmatrix} \qquad D = \begin{bmatrix} 3 & -1 \\ -6 & 2 \end{bmatrix} \qquad D^{-1} = \begin{bmatrix} & \\ & \end{bmatrix}$$

How do we find the inverse of a 3 x 3?

Inverse of a 3 x 3 Matrix:

$$\begin{vmatrix} 1 & 0 & 0 & \vdots & 1 & 0 & 0 \\ 3 & 4 & 0 & \vdots & 0 & 1 & 0 \\ 4 & 5 & 5 & \vdots & 0 & 0 & 1 \end{vmatrix}$$

Matrix Equation:

4x – 2y = 6
2x + 3y = -8

$$\rightarrow \begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \end{bmatrix}$$

*Order will matter with matrices
1.) Find the inverse of the coefficient matrix
2.) Multiply by the inverse on both sides **(*inverse goes first)**
3.) Answer is an ordered pair.

Ex- Solve the following systems using a matrix equation:

1. $\begin{cases} 2x + 4y = 10 \\ x - 3y = -5 \end{cases}$

2. $\begin{cases} 2x - 3y + z = 0 \\ x + y - 2z = 5 \\ -2x + 2y + 4z = 2 \end{cases}$

Area of a Triangle: A= $\pm \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ where the $\pm$ is used to get positive area.

Ex - Find the area of the triangle (1, -1) (4, 3) (0, 5)

Ex - Find the area of the triangle (4, -2) (7, 9) (1, -5)

A= $\pm \frac{1}{2} \begin{vmatrix} 1 & -1 & 1 \\ 4 & 3 & 1 \\ 0 & 5 & 1 \end{vmatrix}$

Test for Collinear Points: A= $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

Ex: Determine whether (-3, -5) (6, 1) (10, 2) are collinear.

Cramer's Rule:

ax + by = e

eliminate y:

eliminate x:

cx + dy = f

$$x = \frac{de - bf}{ad - bc} \qquad y = \frac{af - ce}{ad - bc}$$

Notice: **ad - bc** = $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$

Ex- $2x - 3y = 9$

$x + 5y = -2$ $\qquad x = \dfrac{\begin{vmatrix} 9 & -3 \\ -2 & 5 \end{vmatrix}}{\begin{vmatrix} 2 & -3 \\ 1 & 5 \end{vmatrix}} = \qquad y = \dfrac{\begin{vmatrix} 2 & 9 \\ 1 & -2 \end{vmatrix}}{\begin{vmatrix} 2 & -3 \\ 1 & 5 \end{vmatrix}} =$

Ex- $2x - y = 8$
$4x - 2y = 16$

Ex- $x + y = 5$
$3x - 2y = 20$

x =

y =

Ex- $5x - 3y = 19$
$7x + 2y = 8$

Ex- $-x + y = 5$
$2x + 4y = 38$

Cramer's $4x + 2y - z = 5$
$2x + y - 5z = -11$
$x - 2y + 3z = 6$

$$x = \frac{\begin{vmatrix} 5 & 2 & -1 \\ -11 & 1 & -5 \\ 6 & -2 & 3 \end{vmatrix}}{\begin{vmatrix} 4 & 2 & -1 \\ 2 & 1 & -5 \\ 1 & -2 & 3 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} 4 & 5 & -1 \\ 2 & -11 & -5 \\ 1 & 6 & 3 \end{vmatrix}}{\begin{vmatrix} 4 & 2 & -1 \\ 2 & 1 & -5 \\ 1 & -2 & 3 \end{vmatrix}} \qquad z = \frac{\begin{vmatrix} 4 & 2 & 5 \\ 2 & 1 & -11 \\ 1 & -2 & 6 \end{vmatrix}}{\begin{vmatrix} 4 & 2 & -1 \\ 2 & 1 & -5 \\ 1 & -2 & 3 \end{vmatrix}}$$

Solve for y:
Ex- $3x - 2y + 2z = -2$
$x - 3y + z = -2$
$2x - y + 4z = 7$

Solve for c:
Ex- $a + 2b - c = -7$
$2a + 3b + 2c = -3$
$a - 2b - 2c = 3$

Solve using matrices:

Ex- $x + y + z = 1$
$2x - y = 0$
$-3x + z = 0$

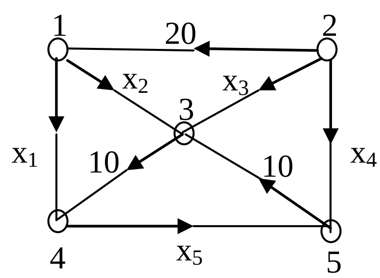
Ex- $a + 2c = -5$
$b - c = 6$
$3a + 2b + c = 3$

Ex - $2x - y + z = 3$
$3x + 2y - 4z = 23$
$x - 3y - 2z = 14$

Ex- $x + 2y = 8$
$3x - y + 2 = -1$
$-2x + 3y - 2z = 10$

Analysis of Network:

In a network, it is assumed that the total flow into a junction is equal to the total flow out of the junction. For instance, junction one has 20 units flowing in so $x_1 + x_2 = 20$.



Set up a system of equations and solve for the unknown values.

Solve the matrix equation for X:

$$\begin{bmatrix} 7 & -2 \\ -4 & 1 \end{bmatrix} X - \begin{bmatrix} 3 & -5 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 2 & -3 \end{bmatrix}$$